unit-1 19-07-2024

simple stresses and strain

mechanical properties of a material

the following are the most important mechanical properties of engineering materials.

1) Elasticity

1) when external forces are applied on a body made of engineering materials, The external forces Temt to Deform The body. The deformation of The particles contineous till full resistance To The external forces is set up.

2) If The forces are now gradually decreases The body build return to it's orginal shape.

Defination : Elasticity is The property of which a material deformed under The load is enabled to return to it's orginal dimension when The load is remove.

EX: steel, copper, allumінium, rubber, spozz

2) Plasticity

1) It is The converse of elasticity it is the property of The material where where The material has a permanent deformation after the stress is removed.

2) A material in Plastic state is permanently deformed by The application of load and it has no

tendency to recover. it is usefull in the design of structural members.

3) Ductility

1) It is a physical property of a material that describes it's ability to be are compressed, pulled are drawn into a thine wire without breaking are ruptured.

2) Ductility is measured in the tensile test of specin one of the material. It is expressed in terms of percentage elongation or percentage reduction in the dimension of the specimen.

4) Brittleness

1) It describes brittleness the property of a material that fractures when subjected to stress but has a little tendency to deform before ruptured.

2) It has high compressive strength and low tensile strength.

EX: cast iron, high carbon steel, concrete, stone, glass, syramic materials.

5) Toughness

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it is a property of a material which enables two absorb energy without fracture. it is measured in terms of energy required per unit volume of the material. under the action of gradually increasing tensile load.

6) hardness

It is the property of a material in which the internal resistance of a material will be developed to localized plastic deformation.

Ex: Iron rods, steel

7)

stress

The internal resistance developed in a body per unit area due to externally applied forces or loads is known as stress.

$$\text{stress} = \frac{\text{External force / load}}{\text{cross sectional area}}$$

$$\sigma = \frac{P}{A} = \frac{N}{mm^2}, \frac{kN}{m^2}$$

Types of stresses

mainly the stresses are classified into two types

1) simple stresses

2) normal stresses

1) simple stresses

1) The stresses developed in a body per unit area due to axial load are known as simple stresses.

2) The stresses are uniformly constant through out the section is known as state of simple stress.

$$\sigma = \frac{P}{A} = \frac{\text{Axial load}}{\text{c/s area}}$$

3) when the load or the resultant of axial load passes through the centroid of the cross section, the stress on a cross section shall be uniform.

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2) Normal Stresses

The internal resistance developed in a material per unit area due to external applied forces act in any direction, mainly two directions.

1) normal to the section

2) Tangential to the section

=> when a body is subjected to axial force acting normal to the surface. Then such forces are known as normal forces or direct forces.

1) Tensile stresses

2) compressive stress

Tensile stresses

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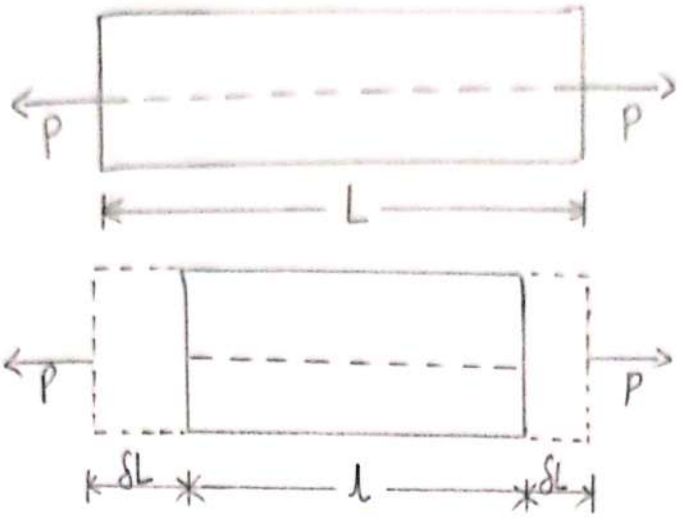
1) consider a straight bar of length "L" and of uniform cross sectional area "a" subjected to a pair of axial forces "P" acting in opposite direction and coinsiding with the axis of the bar.

2) when the applied forces are directed away from the bar that forces are known as Tensile forces and the bar is said to be subjected to pull the bar tends to increase in length.

3) The stresses induced in the section to axial pulls "P" acting normally across the section are known as Tensile stresses.



Tensile stresses = $\frac{\text{Tensile force}}{\text{cross sectional area}} = \frac{P}{A}$ N/mm^2 //

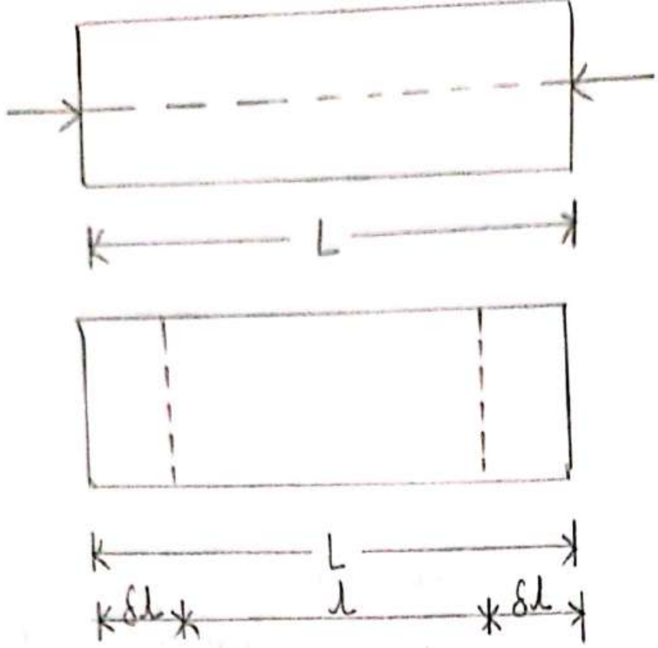
compressive stress

1) consider the straight bar of uniform cross section "a" let the section is subjected to a pair of axial forces "P" acting toward the bar and coinsiding with the axis of the bar.

2) when the applied forces are directed towards the bar that forces are known as compressive forces and the bar is subjected to push. Due to compressive force the bar tends to decrease in length.

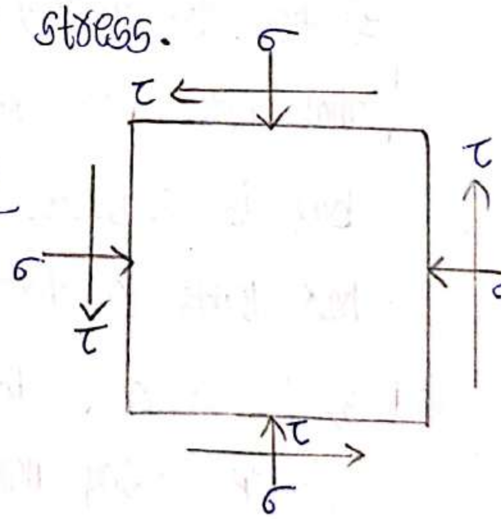
3) the stresses developed in the section to push "P" acting normally across the section are known as compressive stresses.

compressive stress = $\frac{\text{compressive load}}{\text{cross sectional area}} = \frac{P}{A}$



Shear stresses

i) when a body is subjected to tangential force acting tangential direction to the surfaces then such forces are known as shear forces. These shear forces along the tangential direction develop the stresses that stresses are known as shear stress.



$$\text{shear stress "}\tau\text{"} = \frac{\text{shear force}}{\text{c/s area}}$$

Strain

It is defined as the ratio of the change in length to the original length of the bar is known as strain.

$$e = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{\text{change in length} - \delta L}{\text{original length}}$$

$$e = \frac{\delta L}{L}$$

∴ it is dimension less quantity

Types of strains

1) compressive strain

2) Tensile strain

3) volumetric strain

4) shear strain

1) compressive strain

It is defined as the ratio of the decreased length to the original length is called compressive strain.

$$\text{compressive strain} = \frac{\text{decreased length}}{\text{original length}}$$

2) Tensile strain

It is defined as the ratio of the decreased length t increased length to the original length is called as Tensile strain.

$$\text{Tensile strain} = \frac{\text{Increased length}}{\text{original length}}$$

3) volumetric strain

It is defined as the ratio of the change in volume to the original volume is called as volumetric strain.

$$\text{volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

4) shear strain

The strain produced by shear stress is known as shear strain.

I.mp stress-strain diagram for mild steel

1) In Tensile stress standard specimen are taken and subjected to Tensile loads in the universal Testing machine and extensometer shall be fixed on a require guage length of the To measure the exten-sion or elongation of the specimen.

2) The specimen is load a To failure by applying a gradual increase in the load at the slowest rate of machine.

3) A load deformation diagram can be drawn by using the reading of extension at different loads.

4) has the load on the specimen increases - The length of the specimen also increases at the sometime we can absorb that the Diameter of the specimen decreases.

5) The percentage reduction in Diameter shall be Taken as Ductility property.

6) from The readings of loads and deformations find the variation of stress and strain and Plot the draw the stress strain Diagram.

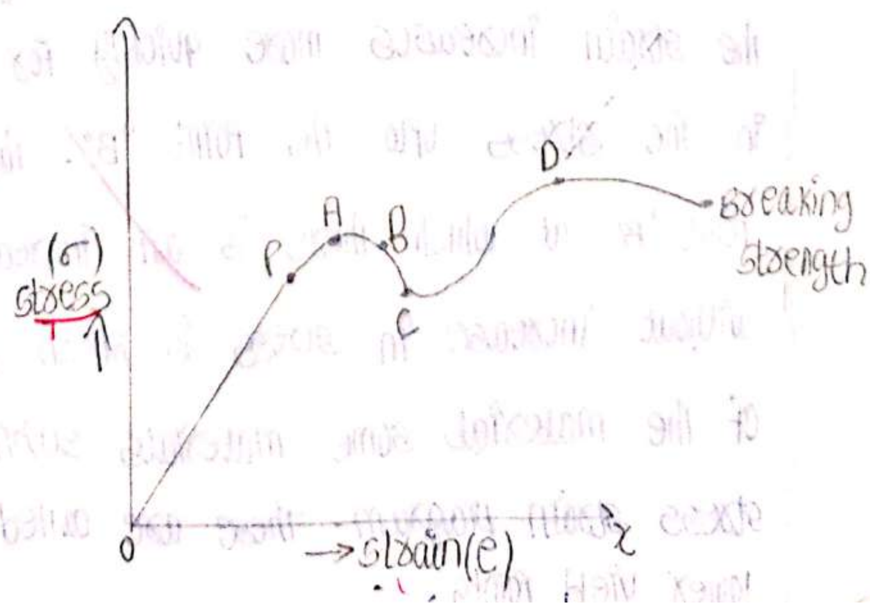
Test specimen

A test specimen used in laboratory two coins "A" and "B" are marked at the centre part of the Test specimen in the above figure note that the end portions of the specimen have increased Diameter which are fixed in two Jaws of the Testing machine. These is Done two see that the failure Takes place only in the central region.

Drawing stress strain Diagram

1) From above observations findout the stresses and strains are calculated upto failure.

2) A stress strain Diagram of mild steel which is a Ductile material is plotted in figure.



where "P" = proportionality limit

A = Elastic limit

"B" = upper yield point

"C" = lower yield point

"D" = ultimate point

properties of material

1) proportionality limit the stress strain curve is a straigh line from the horizon "O" upto certain point "P" is known as proportional limit. beyond these point the strain is longer proportion to the stress.

2) Elastic limit

A material is said to be perfectly elastic if the deformations due to external loadings entirly Disappear on removel of the load. for every material a limiting value of stress is found upto elastic limit.

3) yield point

when a material is located beyond the elastic limit the strain increases more quickly for small increases. in the stress upto the point "B". the ordinate of point "B" at which there is an increase in strain without increase in stress is known as yield point of the material some materials exibit two coins on the stress strain diagram. these are called the upper and lower yield points.

4) Yield strength

The yield load divided by orginal cross sectional area at yield point is known as yield point.

5) Ultimate strength

The ordinate of the point "D" in the stress strain curve is the maximum strength are attained by the material. It is also known as ultimate strength.

6) Breaking strength

The ordinate of point "E" in the stress-strain curve represents the stress at failure and is known as breaking strength are ruptured strength.

Hook's law

It states that for a material subjected to simple tension or compression within elastic limit, the stress is directly proportional to the strain.

stress $\propto$ strain

$$\sigma \propto e$$

$$\sigma = Ee$$

$$\frac{\sigma}{e} = E$$

E = young's modulus of Elasticity (or) modulus of Elasticity is defined as the ratio of compressive stresses or tensile stresses to strain is known as young's modulus.

change in length of a body

consider a bar subjected to a pull.

let length l = original length of the bar

δl = change in length

A = cross sectional area of the bar

P = axial load

from hook's law

$$\frac{\sigma}{e} = E$$

$$\frac{(P/A)}{(\delta l / l)} = E$$

$$\frac{P}{A} \times \frac{l}{\delta l} = E$$

$$\Rightarrow \boxed{\delta l = \frac{Pl}{AE}}$$

25/07/2024

working stress

The maximum stress to a material can be stressed to work with it safety is called working stress/permissib stress/ average stress. it is much less than the proportional limit and well within elastic limit.

factor of safety

it is defined has the ratio of ultimate tensile stress to the working stress.

$$f.o.s = \frac{\text{yield stress}}{\text{permissible stress}}$$

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longitudinal strain (or) linear strain (or) lateral strain

it is the ratio of change in length to the original length of the body is known as linear strain.

$$e_{lo} = \frac{\delta l}{l}$$

Lateral strain

It is the ratio of change in lateral dimension to the original dimension.

(or)

The strain at right angles to the direction of applied load is known as lateral strain.

$$e_{la} = \frac{\delta l}{l} \;/\; e_{la} = \frac{\delta d}{d}$$

Passion's ratio "μ"

The ratio of lateral strain to the longitudinal strain for a given material is known as Passion's ratio.

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

problems

1) A rod 150cm long and of diameter 2cm is subjected an axial pull of 20kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm². Determine stress, strain and elongation.

sol) Given data

length of rod "l" = 150 cm = 1500mm

Diameter of rod "D" = 2cm = 20mm

axial load "P" = 20kN = 20×10^3 N modulus of Elasticity E = 2×10 N/mm²

(i) stress

$$\text{stress } \sigma = \frac{P}{A} = \frac{20\times10^3}{\frac{\pi}{4}\times d^2}$$

$$= \frac{20\times10^3}{\frac{\pi}{4}\times 20^2} = 63.66\ N/mm^2$$

(ii) strain

$$\text{strain } e = \frac{\sigma}{E} = \frac{63.66}{2\times10^5} = 0.0003$$

(iii) Elongation

$$\delta l = \frac{Pl}{AE} = \frac{20\times10^3\times1500}{\frac{\pi}{4}\times20^2\times2\times10^5} = 0.477\ mm$$

Find the minimum Diameter of a steel wire which is used to raise a load of 4000N If the rod is not to exceed 95 MN/m².

Given data

$$\text{axial load "P"} = 4000N$$

$$\text{stress } \sigma = 95\ MN/m^2$$

$$= 95\times10^6\ N/m^2$$

$$= 95\ N/mm^2$$

$$\text{stress } \sigma = \frac{P}{A}$$

$$95 = \frac{4000}{\frac{\pi}{4}\times d^2}$$

$$d^2 = \frac{4000\times4}{\pi\times95}$$

$$d = 7.32\ mm$$

3) A tensile test was conducted on a mild steel bar. The following data was uplend for the test.

1) Diameter of steel pipe 3cm, guage length of the 20cm load at elastic limit 250kN. extension at a load of 150kN, 0.21mm maximum load 380kN. Total extension = 60mm, diameter of the rod at failure = 2.25cm.

1) Determine young's modulus.

2) The stress at elastic limit

3) The percentage elongation

4) The percentage decrease in area.

Sol) Given data

Diameter of bar "D" = 3cm = 30mm

length of bar "l" = 20cm = 200mm

axial load "P" = 250kN = 250×10^3 N

change in length "δl" = 0.21mm

maximum load "W" = 380 kN = 380×10^3 N

Total extension = 60mm

Diameter of rod at failure = 2.25cm = 22.5mm,,

(i) young's modulus

Area of rod "A" = $\frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times 30^2$

$= 706.85 mm^2$

stress "σ" = $\frac{P}{A} = \frac{150 \times 10^3}{706.85} = 212.20 N/mm^2$

strain "e" $= \frac{\delta L}{L} = \frac{0.21}{200} = 1.05\times10^{-3}$

(i) youngs modulus "E" $= \frac{\sigma}{e} = \frac{212.20}{1.05\times10^{-3}} = 2.02\times10^{5}\ N/mm^2$

(ii) the stress at elastic limit

stress "σ" $= \frac{P}{A} = \frac{250\times10^{3}}{706.85} = 353.69\ N/mm^2$

(iii) percentage elongation

$$= \frac{60}{200}\times100 = 30\%.$$

(iv) percentage decrease in area

$$= \frac{-\text{final area} + \text{original area}}{\text{original area}}\times100$$

$$= \frac{-\pi/4\times(22.5)^2 + \pi/4\times(30)^2}{\pi/4\times30^2}\times100$$

$$= \frac{30^2-22.5^2}{30^2}\times100$$

$$= 43.75\%.$$

4) The safe stress for a hallow steel column which carries an axial load of 2.1×10^3 kN is 125 mN/m². if the external diameter of the column is 30cm. Determine the internal diameter.

sol) Given data

safe stress of steel column (σ) $= 125\ mN/m^2$

$= 125\ N/mm^2$

Axial load "p" $= 2.1\times10^3\ kN = 2.1\times10^6\ N$

External Diameter of steel "D" = 30cm = 300mm

Area of steel column "A" $= \frac{\pi}{4}(D^2-d^2)$

$= \frac{\pi}{4}(300^2-d^2)$

stress $\sigma = \frac{P}{A}$

$$125 = \frac{2.1\times10^6}{\frac{\pi}{4}(300^2-d^2)}$$

$$300^2-d^2 = \frac{2.1\times10^6\times4}{125\times\pi}$$

$$300^2-d^2 = 21390.42$$

$$-d^2 = 21390.42-300^2$$

$$\cancel{-}d^2 = \cancel{-}68609.57$$

Interal Diameter $\boxed{d = 261.93mm}$,,

5) Find the youngs modulus of a brase rod of Diameter 25mm and of length 250mm which is subjected to a tensile load of 50kN. when the extension of the rod is 0.3mm.

sol) Given data

Diameter of brase rod "d" = 25mm

length of brase rod "l" = 250mm

axial load "P" = 50kN = 50×10^3N

Extension ~~load~~ of rod δl = 0.3mm

stress "σ" $= \frac{P}{A} = \frac{50\times10^3}{\pi/4\times25^2} = 101.85$ N/mm²

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strain "e" = $\frac{\delta l}{l} = \frac{0.3}{250} = 1.2\times10^{-3}$

young's modulus

$$E = \frac{\sigma}{e} = \frac{101.85}{1.2\times10^{-3}} = 84.87 N/mm^2$$

(6)

26/07/2024

The ultimate stress for a hallow steel column which carries an axial load of 1.9mN is 480N/mm². If the external diameter of column is 200mm, Determine the internal Diameter. Take the factor of safety "4".

Sol) Given data

axial load "P" = 1.9mN

$= 1.9\times10^6 N$

Diameter of external "D" = 200mm

ultimate strength steel column = 480 N/mm²

factor of safety = 4

$$\text{factor of safety} = \frac{\text{ultimate stress}}{\text{permissible stress}}$$

$$4 = \frac{480}{P.S}$$

$$\text{permissible stress} = \frac{480}{4} = 120 N/mm^2$$

$$\sigma = \frac{P}{A}$$

$$120 = \frac{1.9\times10^6}{\frac{\pi}{4}(D^2-d^2)}$$

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$$120 = \frac{1.9\times10^6}{\frac{\pi}{4}(200^2-d^2)}$$

$$(200^2-d^2) = \frac{1.9\times10^6\times4}{120\times\pi}$$

$$(200^2-d^2) = 20159.62$$

$$-d^2 = 20159.62 - 200^2$$

$$\cancel{-}d^2 = \cancel{-}1984038$$

$$d = 140.85mm$$

Elastic constants

Bulk modulus (k)

The ratio of direct stress to the corresponding volumetric strain to be constant for a given material

$$k = \frac{\sigma}{e_v} = \frac{\sigma}{\frac{\delta v}{v}} = N/mm^2$$

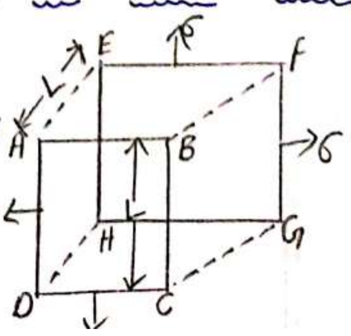
modulus of rigidity (G or C)

The ratio of the shear stress to the shear strain of a given material is known as modulus of rigidity.

$$G \text{ or } C = \frac{\tau}{\phi} = N/mm^2.$$

Expression for relation b/w bulk modulus and youngs modulus

A cube A,B,C,D - E,F,G,H whis is subjected to 3 mutually perpendicular tensile stresses of equal intensity.



let "L" = length of the cube

"E" = young's modulus of the material

"σ" = Tensile stress

"μ" = Possion's ratio

dL (or) δL = change in length of the cube

δv = change in volume of the cube

$\Rightarrow$ now let us consider the strain of one of the sides of cube under the action of the three mutually perpendicular stresses.

① strain of AB due to stresses on the faces AEDH and BFGC these strain is Tensile "e" $= \frac{\sigma}{E}$

② strain of AB due to stresses on the faces AEFB, and BHCG these is compressive

lateral strain $= -\mu \frac{\sigma}{E}$

③ strain of AB due to stresses on the faces ABCD and EFGH. These is also compressive lateral strain $= -\mu \frac{\sigma}{E}$

the Total strain AB

$$e = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\frac{\delta L}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1 - \mu - \mu)$$

$$\frac{\delta L}{L} = \frac{\sigma}{E}(1 - 2\mu) \rightarrow ①$$

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Volume of the cube $V = L \times L \times L = L^3 \rightarrow (2)$

Differentiating above equation with respect to "L"

$$V = L^3$$

$$\frac{dV}{dL} = 3L^2$$

$$dV = 3L^2\, dL \rightarrow (3)$$

$$\frac{(3)}{(2)} = \frac{dV}{V} = \frac{3L^2\, dL}{L^3}$$

$$\frac{dV}{V} = 3\left(\frac{dL}{L}\right)$$

$$\frac{dV}{V} = 3\left(\frac{\sigma}{E}(1-2m)\right)$$

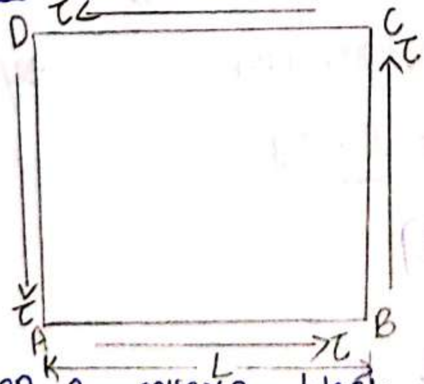
$$\boxed{\text{Bulk modulus } K = \frac{\sigma}{e_V} = \frac{\sigma}{\left(\frac{dV}{V}\right)}}$$

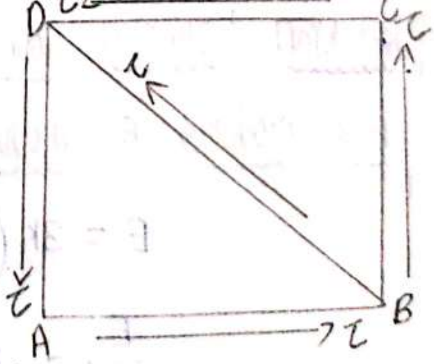
$$= \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$K = \frac{E}{3(1-2m)}$$

$$\boxed{3K(1-2\mu) = E}$$

Expression for relation b/w modulus of rigidity and modulus of elasticity





1) When a square block of unit thickness is subjected to a set off shear stresses of magnitude "τ". on faces AB, CD and AD & CB. The Diagonal strain due to shear stress will be developed.

i.e shear strain ϕ

2) Tensile strain along the diagonal $BD = \frac{\tau}{E}$

$$AC = \mu \frac{\tau}{E}$$

total Tensile strain along BD diagonal

$$= \frac{\tau}{E} + \mu \frac{\tau}{E}$$

$$= \frac{\tau}{E}(1+\mu) \rightarrow (1)$$

Tensile strain along diagonal $BD = \frac{1}{2} \times$ shear strain

$$= \frac{1}{2} \times \phi$$

$$= \frac{1}{2} \times \frac{\text{shear strain}}{\text{modulus of rigidity}}$$

$$= \frac{1}{2} \times \frac{\tau}{G} \rightarrow (2)$$

equating (1) & (2) equations

$$\frac{\cancel{\tau}}{E}(1+\mu) = \frac{1}{2} \times \frac{\cancel{\tau}}{G}$$

$$\boxed{2G(1+\mu) = E}$$

<u>Relation between youngs modulus and Bulk modulus & modulus of rigidity</u>

29/07/2024

$$E = 3K(1-2\mu)$$

$$\frac{E}{3K} = 1-2\mu$$

$$\frac{E}{3K} - 1 = -2\mu$$

$$\cancel{+}\frac{1}{2}\left[1-\frac{E}{3K}\right]=\cancel{+}\mu$$

$$\mu=\frac{1}{2}-\frac{E}{K} \longrightarrow (1)$$

$$E=2G(1+\mu)$$

$$\frac{E}{2G}=1+\mu$$

$$\mu=\frac{E}{2G}-1 \longrightarrow (2)$$

equating (1) and (2)

$$\frac{1}{2}-\frac{E}{6K}=\frac{E}{2G}-1$$

$$\frac{1}{2}+1=\frac{E}{6K}+\frac{E}{2G}$$

$$\frac{3}{2}=\frac{E}{2}\left[\frac{1}{3K}+\frac{1}{G}\right]$$

$$3=E\left(\frac{G+3K}{3KG}\right)$$

$$\frac{9KG}{3K+G}=E$$

Problems

1) for a material youngs modulus is given as $1.2\times10^{5}\,N/mm^2$ and possion's ratio $\frac{1}{4}$. calculate the Bulk modulus.

sol) Given data

Youngs modulus "E" $=1.2\times10^{5}\,N/mm^2$

Possion's ratio "μ" $=\frac{1}{4}$

Bulk modulus "K" $=?$

$$E = 3K(1-2\mu)$$

$$1.2 \times 10^5 = 3 \times K(1-2 \times 0.25)$$

$$K = 80 \times 10^3 \, N/mm^2$$

2) A bar of 30mm diameter is subjected to a pull of 60kN. The measured extension on a guage length of 200mm is 0.1mm and change in diameter is 0.004 mm. calculate i) young's modulus ii) poission's ratio iii) Bulk modulus.

Sol) Given data

Diameter of bar "D" = 30mm

axial load "P" = $60 \times 10^3 N$

guage length "l" = 200mm

change in length "δl" = 0.1mm

change in Diameter "d" = 0.004mm

i) young's modulus (E)

$$E = \frac{\sigma}{e}$$

$$\sigma = \frac{P}{A} \quad \frac{60 \times 10^3}{\pi/4 \times (30)^2} = 84.88 \, N/mm^2$$

$$e = \frac{\delta l}{l} = \frac{0.1}{200} = 0.0005$$

$$E = \frac{84.88}{0.0005} = 169760 \, N/mm^2$$

$$E = 1.69 \times 10^3 \, N/mm^2$$

ii) Poisson's ratio (μ)

$$\mu = \frac{\text{lateral strain}}{\text{linear strain}}$$

$$= \frac{\delta d/d}{\delta l/l} = \frac{0.004/30}{0.1/200} = 0.267$$

iii) Bulk modulus (K)

$$E = 3K(1-2\mu)$$

$$1.69 \times 10^3 = 3 \times K(1-2 \times 0.267)$$

$$K = 120.88 \times 10^3 \, N/mm^2$$

3) Determine the poisson's ratio and Bulk modulus of material for which youngs modulus is $1.2 \times 10^5 \, N/mm^2$ and modulus of rigidity is $4.8 \times 10^4 \, N/mm^2$.

sol) Given data

youngs modulus "E" = $1.2 \times 10^5 \, N/mm^2$

modulus of rigidity "G" = $4.8 \times 10^4 \, N/mm^2$

Poisson's ratio "μ" $= \frac{E}{2G} - 1 = \frac{1.2 \times 10^5}{2(4.8 \times 10^4)} - 1$

$= 0.025$

Bulk modulus "K" $= \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^5}{3 \times (1-2(0.025))}$

$= 42105.2 \, N/mm^2$

$= 42.105 \times 10^3 \, N/mm^2$

Analysis of Bars of varying sections: 31/07/2024

A bar of different lengths and of different Diameters. let this bar is subjected to an axial load "P".

=> through each section is subjected to the same axial load "P" yet the stresses strains and change in length will be different. The total change in length will be obtained by adding the changes in length of individual sections.

=> strain for the sections

section - ① section - ② section - ③

$$e_1 = \frac{\sigma_1}{E} \qquad e_2 = \frac{\sigma_2}{E} \qquad e_3 = \frac{\sigma_3}{E}$$

=> stresses for the sections

section - ① section - ② section - ③

$$\sigma_1 = \frac{P}{A_1} \qquad \sigma_2 = \frac{P}{A_2} \qquad \sigma_3 = \frac{P}{A_3}$$

change in length / elongation of the bar at the sections.

section - ① $\delta l_1 = \dfrac{P l_1}{A_1 E}$

section - ② $\delta l_2 = \dfrac{P l_2}{A_2 E}$

section - ③ $\delta l_3 = \dfrac{P l_3}{A_3 E}$

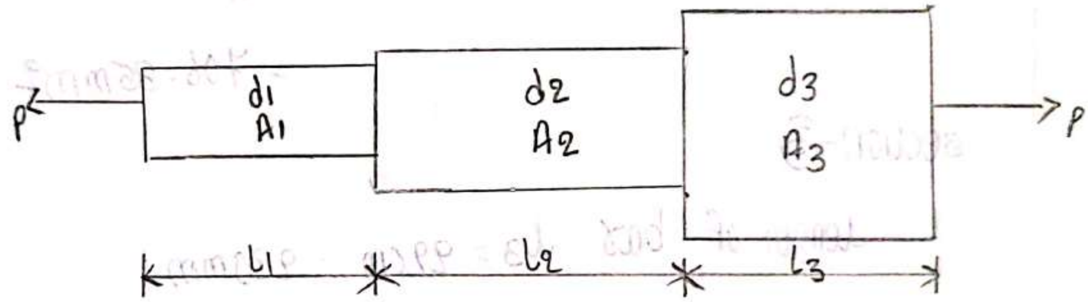
Total elongation of the bar $(\delta L) = \delta L_1 + \delta L_2 + \delta L_3$

$$= \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\delta L = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

For different materials. Young's modulus will be different.

$$\delta L = P\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right)$$ //



<u>Problems</u>

1) An axial pull of 35000N is acting on a bar consisting of 3 lengths as shown in figure. If the Young's modulus 2×10^5 N/mm². Determine i) stresses in each section

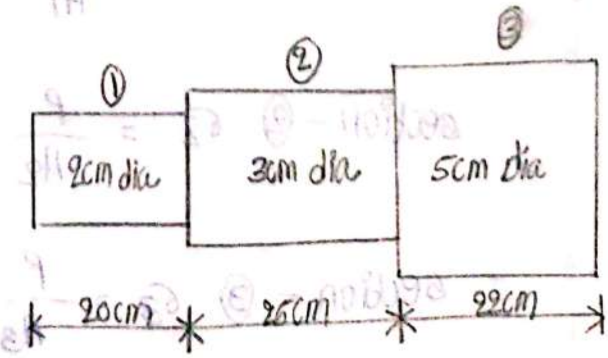
ii) Total elongation of the bar.

Sol) <u>Given data</u>

axial load "P" = 35×10^3 N

Young's modulus "E"

= 2×10^5 N/mm²



section - 1

Length of bar $l_1 = 20cm = 200mm$

Diameter of bar $d_1 = 2cm = 20mm$

Area of bar $A_1 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 20^2$

$= 314.15 mm^2$

section - 2

Length of bar $l_2 = 25cm = 250mm$

Diameter of bar $d_2 = 3cm = 30mm$

Area of bar $A_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 30^2$

$= 706.85 mm^2$

section - 3

Length of bar $l_3 = 22cm = 220mm$

Diameter of bar "d_3" $= 5cm = 50mm$

Area of bar $A_3 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 50^2$

$= 1963.49 mm^2$

i) stresses at each ~~an~~ section

section - 1 $\sigma_1 = \frac{P}{A_1} = \frac{35 \times 10^3}{314.15} = 111.41 N/mm^2$

section - 2 $\sigma_2 = \frac{P}{A_2} = \frac{35 \times 10^3}{706.85} = 49.51 N/mm^2$

section - 3 $\sigma_3 = \frac{P}{A_3} = \frac{35 \times 10^3}{1963.49} = 17.82 N/mm^2$

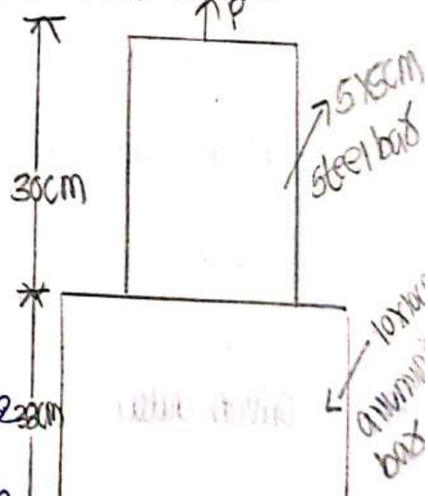
ii) Total elongation

$$\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

$$= \frac{35000}{2.1 \times 10^5}\left(\frac{200}{314.15} + \frac{250}{706.85} + \frac{220}{1963.49}\right)$$

$$= 0.1837 \text{mm}$$

2) A member formed by connecting is steel bar to an alluminium bar is shown in figure. calculate the force magnitude of force "P" That will cause the total length of The member to decrease 0.25mm. The values of elastic modulus for steel and alluminium are 2.1×10^5 N/mm² and 7×10^4 N/mm² respectively.



Sol) Given data

change in length "δl" = 0.25mm

Young's modulus "E_1" = 2.1×10^5 N/mm²

"E_2" = 7×10^4 N/mm²

Length of steel l_1 = 30cm

= 300mm

Area of steel $A_1 = L \times B$

$= 5 \times 5$

$= 25 \text{cm}^2 = 2500 \text{mm}^2$

Total elongation of the bar

$$\delta l = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right]$$

$$0.25 = P\left\{\frac{300}{2500 \times 2.1\times10^5 N/mm^2} + \frac{380}{10,000 \times 7\times10^4 N/mm^2}\right\}$$

$$0.25 = P\left[5.714\times10^{-3} + 5.4285\times10^{7}\right]$$

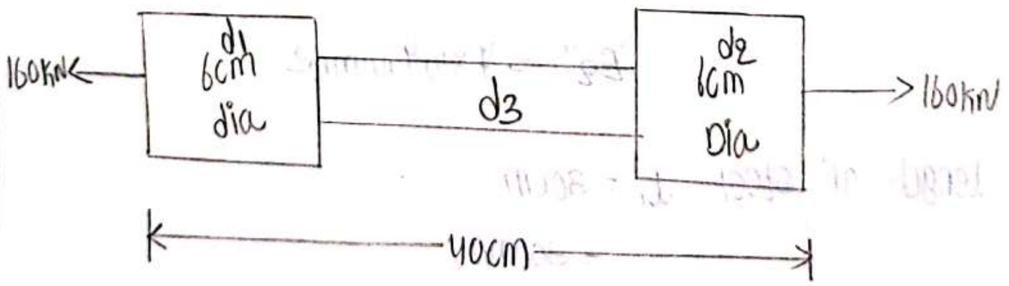
$$0.25 = P\left[1.114\times10^{-6}\right]$$

$$P = 43.75 N/mm^2$$

$$P = 222.4 kN$$

3) A bar shown in figure is subjected to a tensile land load of 160kN, if the stress in the middle portion is 150 N/mm2. Determine the diameter of the middle portion find also the length of the middle portion if the total elongation of the bar is 0.2mm young's modulus $2.1\times10^5 N/mm^2$.

Sol) Given data



Young's modulus "E" = $2.1\times10^5 N/mm^2$

Tensile load "P" = 160kN

Change in length $\delta l = 0.2mm$

stress at middle portion $\sigma_2 = 160 N/mm^2$

(i) Diameter of middle portion $d_2 = ?$

$$\sigma_2 = \frac{P}{A_2} = 160 = \frac{160\times10^3}{\frac{\pi}{4}(d_2)^2}$$

$$d_2^2 = \frac{160\times10^3\times4}{\pi\times150} = 36.85 mm$$

$$A_2 = \frac{\pi}{4}\times36.85^2 = 1066.50 mm^2$$

(ii) Length of middle portion $L_2 = ?$

Length of Bar $L = 40cm = 400mm$

Let $L_1+L_2 = 400$

$L_1 = 400-L_2$

$$\delta L = \frac{P}{E}\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}\right]$$

$$0.2 = \frac{160\times10^3}{2.1\times10^5}\left[\frac{400-L_2}{2827.43}+\frac{L_2}{1066.50}\right]$$

$L_2 = 206.15mm$ $= 20.615cm$

Analysis of Bars of composite sections:

A bar made of two or more bars of equal lengths but of different materials fixed with each other and behaving as one unit for extension or compression when subjected to an tensile or compressive loads is called a composite bar.

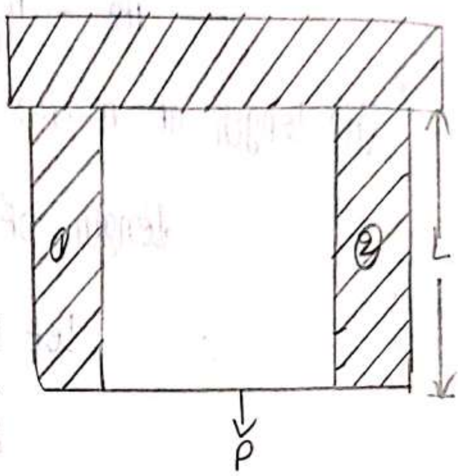
1) The extension or compression in each bar is equal hence strain in each bar is equal.

2) The total external load and the composite bar is equal to the some of the loads carried by each different material.

3) A composite bar made up of two different materials

4) the total load on the composite bar is equal to the sum of the load carried by the two bars.

$$P = P_1 + P_2$$
$$= (\sigma_1 A_1) + (\sigma_2 A_2)$$



5) strain in bar 1, $e_1 = \frac{\sigma_1}{E_1}$

2, $e_2 = \frac{\sigma_2}{E_2}$

In composite bar,

strain in bar ① = strain in bar ②

$$e_1 = e_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

The ratio of $\frac{E_1}{E_2}$ is called modular ratio

problems

1) A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and in diameter 4cm. The composite bar is subjected to an axial pull of 45000N. If the length of the bar is equal to the 15cm. Determine

i) The stresses in the rod & Tube

ii) Load carried by each bar

E for steel = 2.1×10^5 N/mm^2, E for the copper = 1.1×10^5 N/mm^2

Sol) Given data

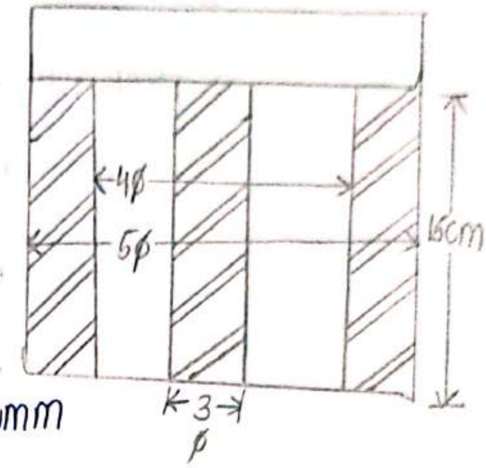
Diameter of steel rod = 3cm

= 30mm

External Diameter of copper, tube = 5cm

= 50mm

Internal Diameter of copper, tube = 4cm = 40mm



axial pull "P" = 45000N

young's modulus for steel "E_s" = 2.1×10^5 N/mm^2

young's modulus for copper "E_c" = 1.1×10^5 N/mm^2

Area of steel Rod, $A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(30)^2$

$= 706.85$ mm^2

Area of copper tube, $A_c = \frac{\pi}{4}(D^2-d^2)$

$= \frac{\pi}{4}(50^2-40^2)$

$= 706.85$ mm^2

length of Bar "L" = 15cm = 150mm

i) Stress in the rod and tube:

strain in steel rod = strain in copper tube

$$e_s = e_c$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$\sigma_s = \frac{2.1\times10^5}{1.1\times10^5}\sigma_c$$

$$\sigma_s = 1.909\sigma_c \rightarrow ①$$

Total load "P" = $P_s + P_c$

$$45000 = \sigma_s A_s + \sigma_c A_c$$

$$45000 = 1.909\sigma_c \times 706.85 + \sigma_c \times 706.85$$

$$45000 = \sigma_c(1.909\times706.85 + 706.85)$$

$$45000 = 2056.22\sigma_c$$

$$\sigma_c = \frac{45000}{2056.22} = 21.88\ N/mm^2$$

from eq ①

$$\sigma_s = 1.909\times\sigma_c = 1.909\times21.88$$

$$= 41.76\ N/mm^2$$

ii) Load carried by each bar :

$$P_s = \sigma_s A_s$$

$$= 41.76\times706.85$$

$$= 29518.56\ N$$

$$= 29.51\ kN$$

Total load, $P = P_s + P_c$

$$P_c = P - P_s$$

$$= 45000 - 29518.056$$

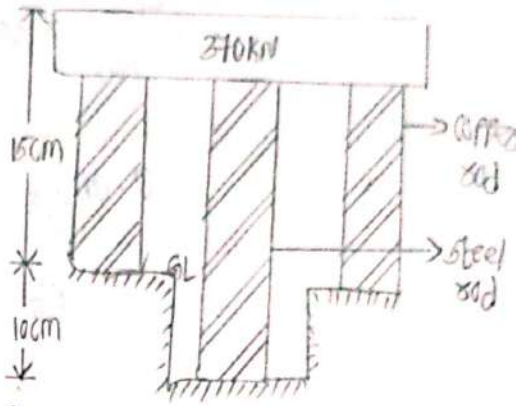
$$= 15481.944\ N$$

$$= 15.48\ kN$$

Scanned with OKEN Scanner

2) A steel rod and two copper rods together supports a load of 370kN as shown in fig. the cross sectional area of steel rod is 2500mm² and for each copper rod is 1600mm². Find the stress in the rod. Take $E_s = 2\times10^5 N/mm^2$ and $E_c = 1\times10^5 N/mm^2$.

Sol) Given data



Total load "P" = 370 kN

C/s area of steel rod,

$A_s = 2500mm^2$

C/s area of each copper rod, $A_C = 1600mm^2 \times 2 = 3200mm^2$

$E_s = 2\times10^5 N/mm^2$

$E_C = 1\times10^5 N/mm^2$

length of copper rod, "L_C" = 15cm = 150mm

length of steel rod "L_s" = 25cm = 250mm

for composite bar,

Total load, $P = P_s + P_c \rightarrow (1)$

strain in steel bar : (1)

$e_s = \frac{\sigma_s}{E_s}$

$\frac{\delta L}{L_s} = \frac{\sigma_s}{E_s}$

$\delta L = \frac{\sigma_s}{E_s} L_s$

strain in copper bar :

$e_c = \frac{\sigma_c}{E_c}$

$\frac{\delta L}{L_c} = \frac{\sigma_c}{E_c}$

$\delta L = \frac{\sigma_c}{E_c} L_c$

In composite Bar,

strain in steel Bar = strain in copper Bar

$$\frac{\sigma_S}{E_S} L_S = \frac{\sigma_C}{E_C} L_C$$

$$\frac{\sigma_S}{2\times10^5} \times 250 = \frac{\sigma_C}{1\times10^5} \times 150$$

$$0.0012\sigma_S = 0.0015\sigma_C$$

$$\sigma_S = \frac{0.0015}{0.0012} \sigma_C$$

$$\sigma_S = 1.25\sigma_C \rightarrow (2)$$

from equation (1)

$$P = P_S + P_C$$

$$370\times10^3 = \sigma_S A_S + \sigma_C A_C$$

$$370\times10^3 = 1.25\sigma_C \times 2500 + \sigma_C \times 3200$$

$$370\times10^3 = \sigma_C (1.25 \times 2500 + 3200)$$

$$370\times10^3 = 6325\sigma_C$$

$$\sigma_C = \frac{370\times10^3}{6325} = 58.49 N/mm^2.$$

$$\sigma_S = 1.25 \times 58.49$$

$$73.11 N/mm^2$$

3) A compound tube consists of a steel tube 140mm in diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter, the 2 tubes are of the same length. The compound tube carries an axial load of 900kN. Find the stress and the load carried by each tube and find the decrease in length. Length of each tube is 140mm. Take $E_s = 2\times10^5 N/mm^2$, $E_b = 1\times10^5 N/mm^2$

Sol) Given data

Steel tube :

Internal Diameter $= 140mm$

External Diameter $= 160mm$

Brass tube

Internal Diameter $= 160mm$

External Diameter $= 180mm$

axial load "P" $= 900\ kN$

Length of each tube $l = 140\ mm$

$E_s = 2\times10^5 N/mm^2$

$E_b = 1\times10^5 N/mm^2$

Area of steel tube, $A_s = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(160^2 - 140^2)$

$= 4712.38mm^2$

Area of brass tube, $A_b = \frac{\pi}{4}(D^2 - d^2)$

$$= \frac{\pi}{4}(180^2 - 160^2)$$

$$= 5340.70 mm^2$$

i) Stress in each tube:

Strain in steel tube = strain in brass tube

$$e_s = e_b$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{E_s}{E_b}\sigma_b$$

$$\sigma_s = \frac{2\times10^5}{1\times10^5}\sigma_b$$

$$\sigma_s = 2\sigma_b \rightarrow ①$$

Total load 'P' = $P_s + P_b \rightarrow ②$

$$900\times10^3 = \sigma_s A_s + \sigma_b A_b$$

$$900\times10^3 = 2\sigma_b \times 4712.38 + \sigma_b \times 5340.70$$

$$900\times10^3 = \sigma_b(2\times4712.38 + 5340.70)$$

$$900\times10^3 = 14765.46\,\sigma_b$$

$$\sigma_b = \frac{900\times10^3}{14765.46}$$

$$\sigma_b = 60.95 N/mm^2$$

from equation ①

$$\sigma_s = 2\sigma_b = 2\times60.95 = 121.9 N/mm^2$$

ii) Load by each tube:

$$P_s = \sigma_s A_s$$

$$= 121.9 \times 4712.38$$

$$= 574439.122 N$$

$$= 574.43 kN$$

from eq ②

$$P = P_s + P_b$$

$$900 \times 10^3 = 574.43 + P_b$$

$$P_b = 900 - 574.43$$

$$P_b = 325.57 kN$$

iii) Decrease in length

$$\delta L = \frac{P_s L}{A_s E_s} = 140 \left[\frac{574.43 \times 10^3}{4712.38 \times 2 \times 10^5} \right]$$

$$= 0.085 mm$$

4) find the young's modulus of a rod of diameter 30mm and off length 300mm which is subjected to a tensile load 60kN and the extension of the rod is equal to 0.4mm.

Sol) Given data

Diameter of rod "d" = 30mm

length of rod "L" = 300mm

Tensile load "P" = 60kN = $60 \times 10^3 N$

Extension of rod "δL" = 0.4mm

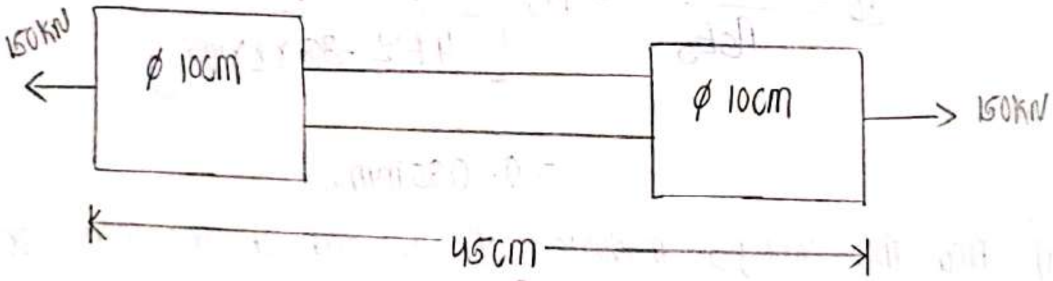
Youngs modulus "E" = ?

$$E = \frac{\sigma}{e} = \frac{P/A}{\delta L/L}$$

$$= \frac{60\times10^3/\frac{\pi}{4}\times30^2}{0.4/300}$$

$$= 63661.97\ N/mm^2$$

5) A bar shown in fig is subjected to a tensile load of 150kN. If the stress in the middle portion is limitted to 160 N/mm². Determine the diameter of the middle portion. also find the length of the middle portion if the total elongation of the bar is to be 0.25cm, $E = 2\times10^5\ N/mm^2$.



sol) Given data

axial load "P" = 150kN = 150×10^3 N

stress in middle portion "σ" = 160 N/mm²

Total elongation of the bar "δL" = 0.25cm
= 2.5mm

Young's modulus "E" = 2×10^5 N/mm²

i) Diameter of middle portion

$$\sigma_2 = \frac{P}{A}$$

$$160 = \frac{150 \times 10^3}{\frac{\pi}{4} \times d^2}$$

$$"d_2" = 34.54 mm$$

ii) Length of middle portion

$$\delta l = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

Length of bar "l" = 45 cm = 450 mm

$$L_1 + L_2 = 450 mm$$

$$L_1 = 450 - L_2$$

$$2.5 = \frac{150 \times 10^3}{2 \times 10^5}\left[\frac{450 - L_2}{\pi/4 \times 100^2} + \frac{L_2}{\pi/4 \times 34.54^2}\right]$$

$$"L_2" = 3485.42 mm$$

Shear force and Bending moment

Shear force

The algebric sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force.

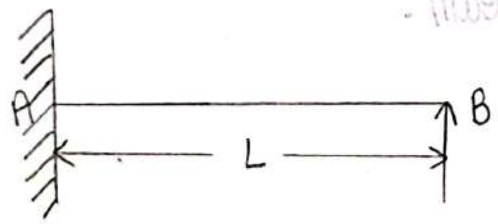
Bending moment

The algebric sum of the moments of all the forces acting to the right or left of the section is known as Bending moment.

Types of Beams

1) Cantilever Beam

A Beam is fixed at one end and free at the other end fixed is called cantilever Beam.



2) Simply supported Beam

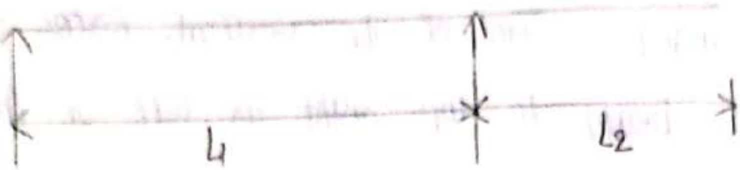
A Beam supported are resting freely on the supports at it's both ends is called simply supported Beam.

3) fixed Beam

Both ends of the Beam will be fixed is called as fixed Beam.

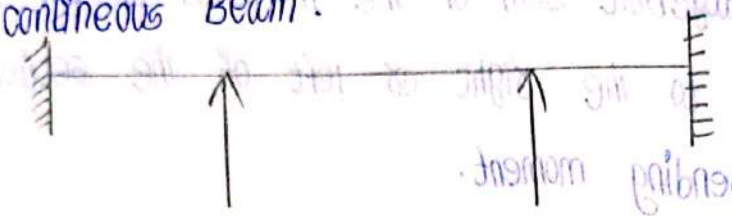
4) over hanging Beam

The end portion of the Beam is extended beyond the support is called as over hanging Beam.



5) contineous Beam

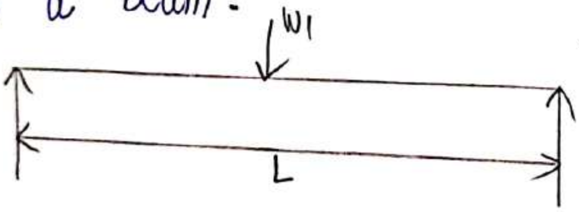
A beam is provided more than two supports is called as contineous Beam.



Types of loads

1) Point load / concentrated load

A load is one which is considered to act at a point on a beam.



2) uniformly Distributed load (UDL)

sign conventions shear force and Bending moment

shear force

The shear force at a section will be considered positive when the resultant of the forces to the left of the section is upward (or) Right of the section is downward.

The shear force at a section will be considered negative if the resultant of the forces to the left of the section is downward (or) to the right of the section is upward.

Bending moment

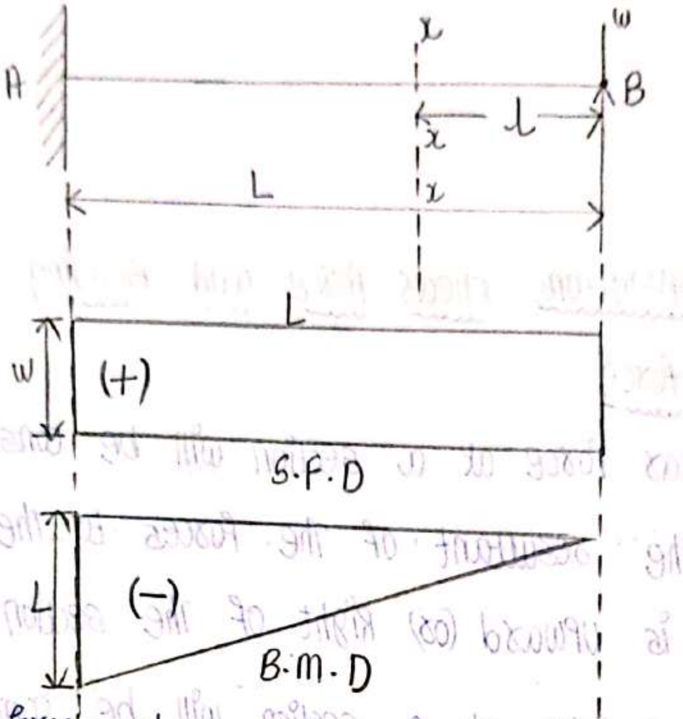
1) The Bending moment at a section is positive if the Bending moment at the section it's tends bend the Beam to a curvature having concavity or concave at the

2) The Bending moment at a section is negative if the Bending moment at the section it's tends to bend the Beam to a curvature having convexity @ the top.

3) The positive Bending moment is called sagging moment

and negative moment has hogging moment.

Shear force and Bending moment diagrams for a cantilever with a point load at the free end



Shear force = at a section x-x is given by

$$F = +W$$

Bending at a section x-x is given by

$$B.M = Wx$$

1) consider a AB of length "L" fixed at end "a" "(A)" and free at the end "b" "(B)" and carrying a point load "w" at the free end "B".

2) Take a section x-x at a distance "x" from the free end the shear force at these section is equal to the resultant force acting on the right portion of the given section.

$$F = +W$$

Bending moment at a section x-x is given by

Bending moment $= -w \times x$

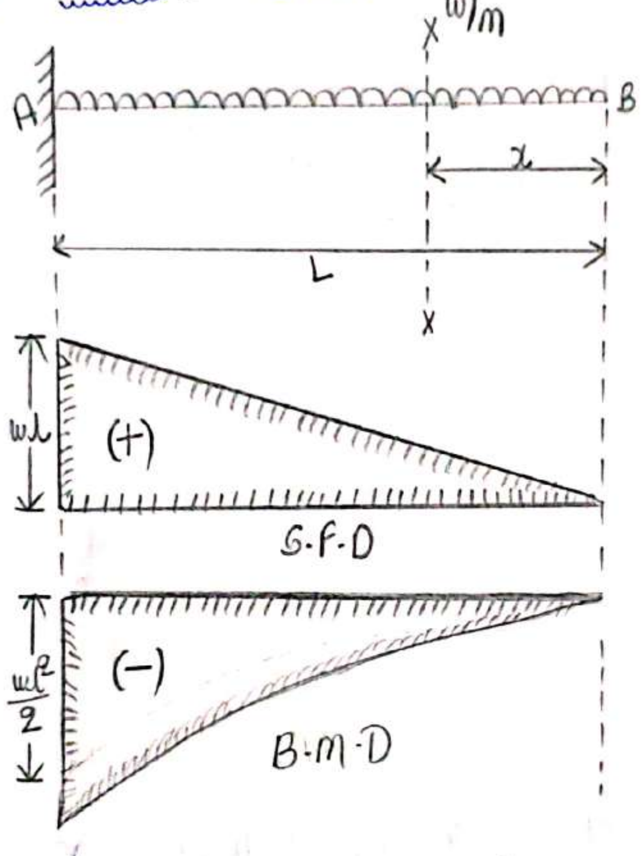
at end "B", $x=0$, B.m $\quad m = -w \times 0$

$= 0$

end "A" $x = L$, B.m $= -w \times L$

$= -wL$, kN-m

Shear force Bending moment Diagram for a cantilever with a uniformly Distributed load



shear force at the section x-x is given by

$F = +w \times x$

At B, $x=0$, $F=0$

At $x = L$; $F = +wL$

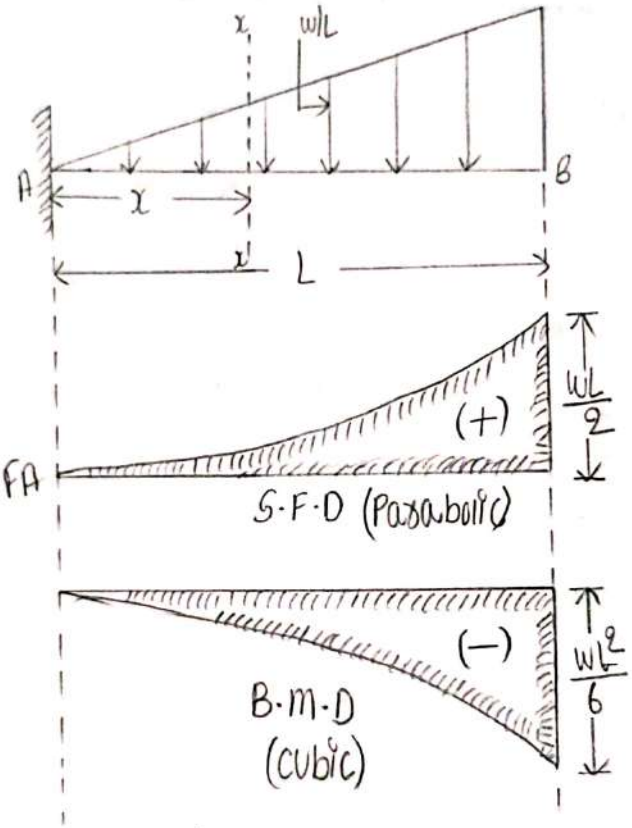
Bending moment at the section x-x

$$B.m = \frac{-wx^2}{2}$$

At B; $x = 0$; $Bm = 0$

A; $x = L$ $B.m = \frac{-wL^2}{2}$

Shear force and Bending moment Diagrams 08/08/2024
for a cantilever carrying gradual varying load



shear force at section x−x @ distance x

$$Fx = \left[\frac{w}{w} \times x\right]$$

Total load = Area of the triangle

$$= \frac{1}{2} \times x \times \frac{wx}{w}$$

$$= + \frac{wx^2}{2L}$$

At A; $x=0$; $F_A = \frac{w \times 0}{2L} = 0$

At B; $x=L$; $F_B = \frac{w \times L^{\cancel{2}}}{2\cancel{L}} = \frac{wL}{2}$

Bending moment

$M_x = -(\text{Total load}) \times \text{centroidal Distance}$

$= \frac{-wx^2}{2L} \times \frac{x}{3}$

$= \frac{-wx^3}{2L}$

At A; $x=0$, $M_A = 0$

At B; $x=L$, $M_B = \frac{-wL^{\cancel{3}}}{6\cancel{L}}$

$= \frac{-wL^2}{6}$

1) A cantilever of length 2m carries a UDL of 2kN/m length over the whole length & a point load of 3kN at the free end. Draw the shear force (S.F) and Bending moment (B.M) diagrams.

Sol) Given data

Shear force : $F_x = W + wx = 3 + 2x$

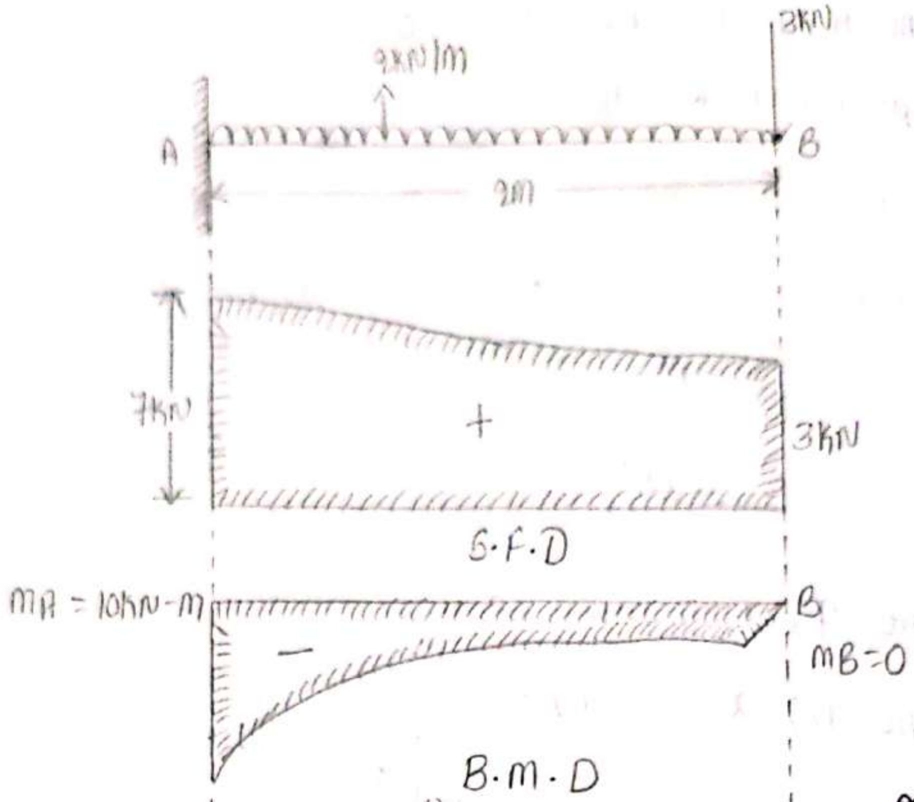
At B; $x=0$, $F_B = 3+0 = 3kN$

At A; $x=2$, $F_A = 3+2\times2 = 7kN$

Bending moment : $M_x = -Wx - \frac{wx^2}{2} = -3x - \frac{2x^2}{2}$

At B; $x=0$, $M_B = 0$

At A; $x=2$, $M_A = -3\times2 - \frac{2\times2^2}{2} = -10kN\text{-}m$



9) Given data

09/08/2024

~~Fx~~ Shear force

$F_x = W$

At D; $F_D = 800N$

$F_x = 800+500 = 1300N$

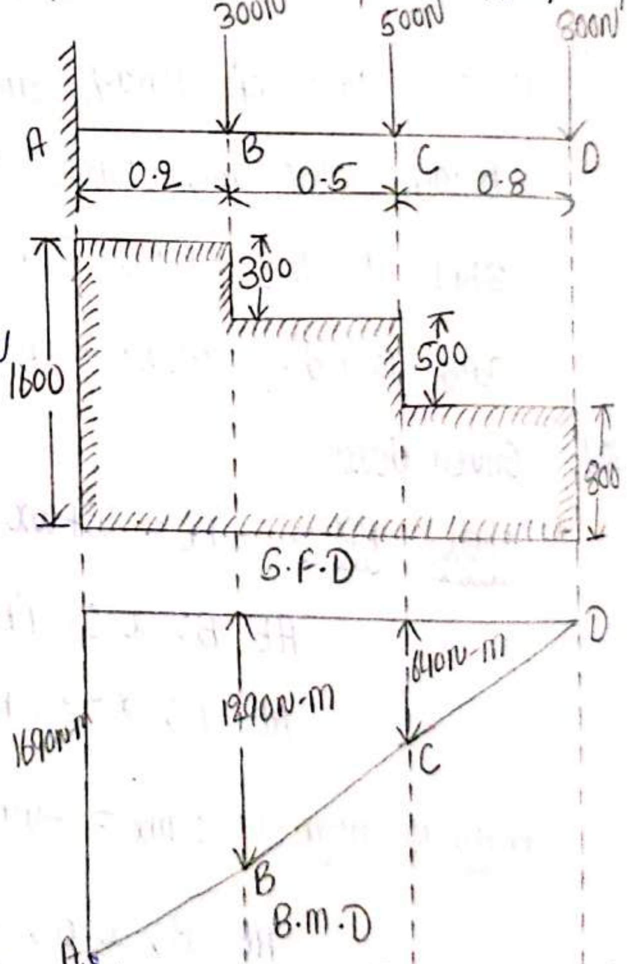
At C; $F_C = 1300N$

$F_x = 800+500+300$

$= 1600N$

At B; $F_B = 1600N$

At A; $F_A = 1600N$



Bending moment

B.m @ section x-x @ a distance of x from D, Bm@D:

$M_x = -wx$

$= -800x$

Boundary condition

At D; $x=0$, $M_D=0$

Bending moment @ B

$$m_x = -wx - w(x-0.8) - w(x-1.3)$$
$$= -800x - 500(x-0.8) - 300(x-1.3)$$

BC

At B ; $x = 1.3m$

$$m_B = -800(1.3) - 500(1.3-0.8) - 300(1.3-1.3)$$
$$= -1290 N\text{-}m$$

At A; $x = 1.5m$

$$m_A = -800(1.5) - 500(1.5-0.8) - 300(1.5-1.3)$$
$$= -1610 N\text{-}m$$

BM @ C

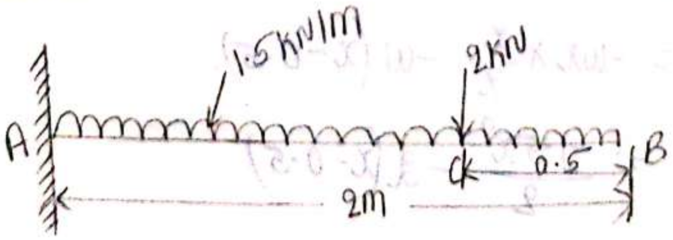
B.m at section x-x portion C and B at a distance of x from D.

$$m_x = -wx - w(x-0.8)$$
$$= -800x - 500(x-0.8)$$

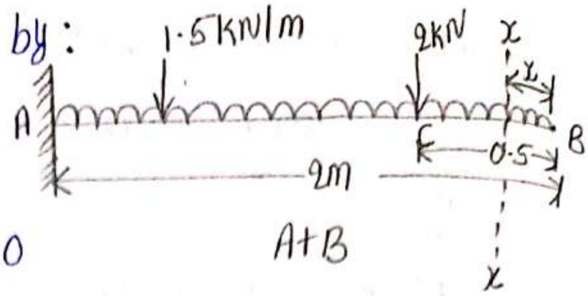
At C; $x = 0.8m$, $m_C = -800 \times 0.8 - 500(0.8-0.8)$
$$= -640 N\text{-}m$$

3) A cantilever of length 2m carries a UDL of 1.5kN/m run over the whole length and a point load of 2kN at a distance of 0.5m from the free end.

sol) Given data



consider X-X section at a position of C and B @ a distance of x is given by:



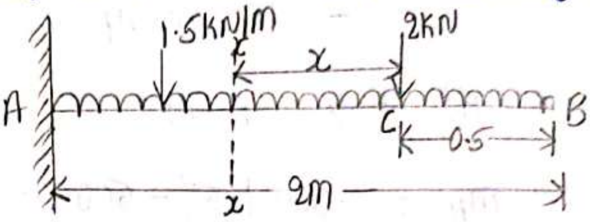
A+B

$F_x = +wx = 1.5x$

At "B"; $x=0$ $F_B = 1.5 \times 0 = 0$

At C; $x = 0.5$, $F_C = 1.5 \times 0.5 = 0.75\,kN$

consider AC portion, locate x-x section @ a distance of x from end B:



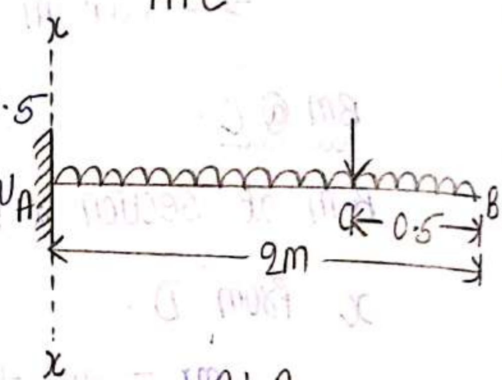
A+C

$F_x = W + wx$

$F_x = 2 + 1.5x$

At C; $x = 0.5\,m$; $F_C = 2 + 1.5 \times 0.5$
$= 2.75\,kN$

At A; $x = 2m$; $F_A = 2 + 1.5 \times 2$
$= 5\,kN$



A+A

Bending moment

i) At C and B portion locate x-x section @ a distance x: 4)

$$m_x = \frac{-wx^2}{2} = \frac{-1.5x^2}{2}$$

At B; $x=0$; $m_B = 0$

At C; $x = 0.5m$; $m_C = \frac{-1.5 \times 0.5^2}{2} = -0.187\,kN\text{-}m$

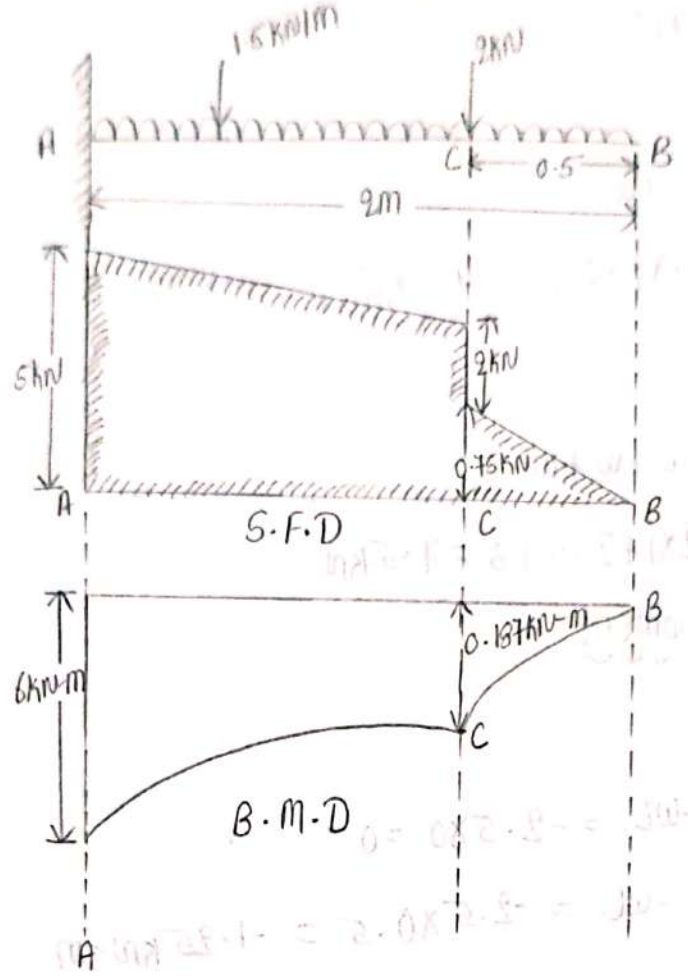
ii) At C and A portion, locate x-x section at a distance of x from B:

$$m_x = -wx \times \frac{x}{2} - W(x - 0.5)$$

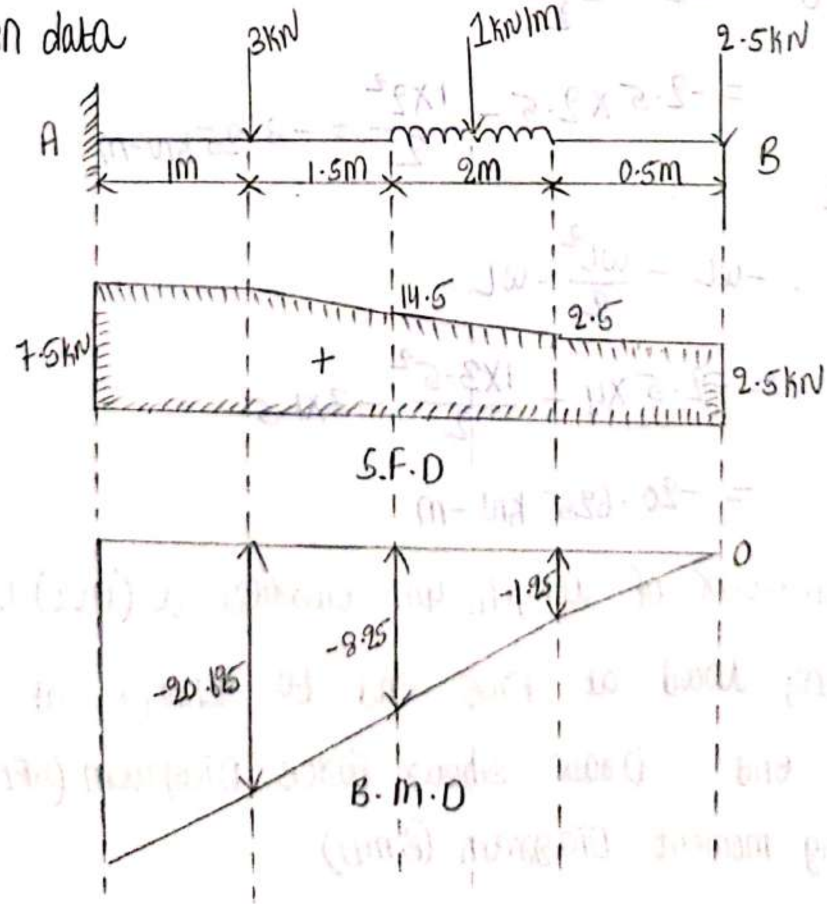
$$= \frac{-1.5x^2}{2} - 2(x - 0.5)$$

At C; $x = 0.5m$, $M_C = \frac{-1.5 \times 0.5^2}{2} - 2(0.5-0.5) = -0.187 \text{ kN-m}$

At A; $x = 2m$, $M_A = \frac{-1.5 \times 2^2}{2} - 2(2-0.5) = -6 \text{ kN-m}$



Given data



Shear force :

portion C & B:

$F_B = 12.5$ kN

C & D

$F_C = +wL + w$

$= +1 \times 2 + 2.5 = 4.5$ kN

D & E

$F_E = +wL + w + w$

$= 2 \times 1 + 2.5 + 3 = 7.5$ kN

Bending moment

C & B

$M_B = -wL = -2.5 \times 0 = 0$

$M_C = -wL = -2.5 \times 0.5 = -1.25$ kN-m

C & D

$$M_D = -wL - \frac{wL^2}{2}$$

$$= -2.5 \times 2.5 - \frac{1 \times 2^2}{2} = -8.25 \text{ kN-m}$$

A & E

$$M_E = -wL - \frac{wL^2}{2} - wL$$

$$= -2.5 \times 4 - \frac{1 \times 3.5^2}{2} - 3 \times 1.5$$

$= -20.625$ kN-m

5) A cantilever of length 4m carries a (UVL) uniform varying load at free end to 2kN/m at the fixed end. Draw Shear Force Diagram (SFD) and Bending moment Diagram (BMD)

Sol) Shear force

At free end B; $F_B = 0$

At fixed end A; $F_A = +\frac{wL}{2}$

$= +\frac{2\times4}{2}$

$= 4 kN$

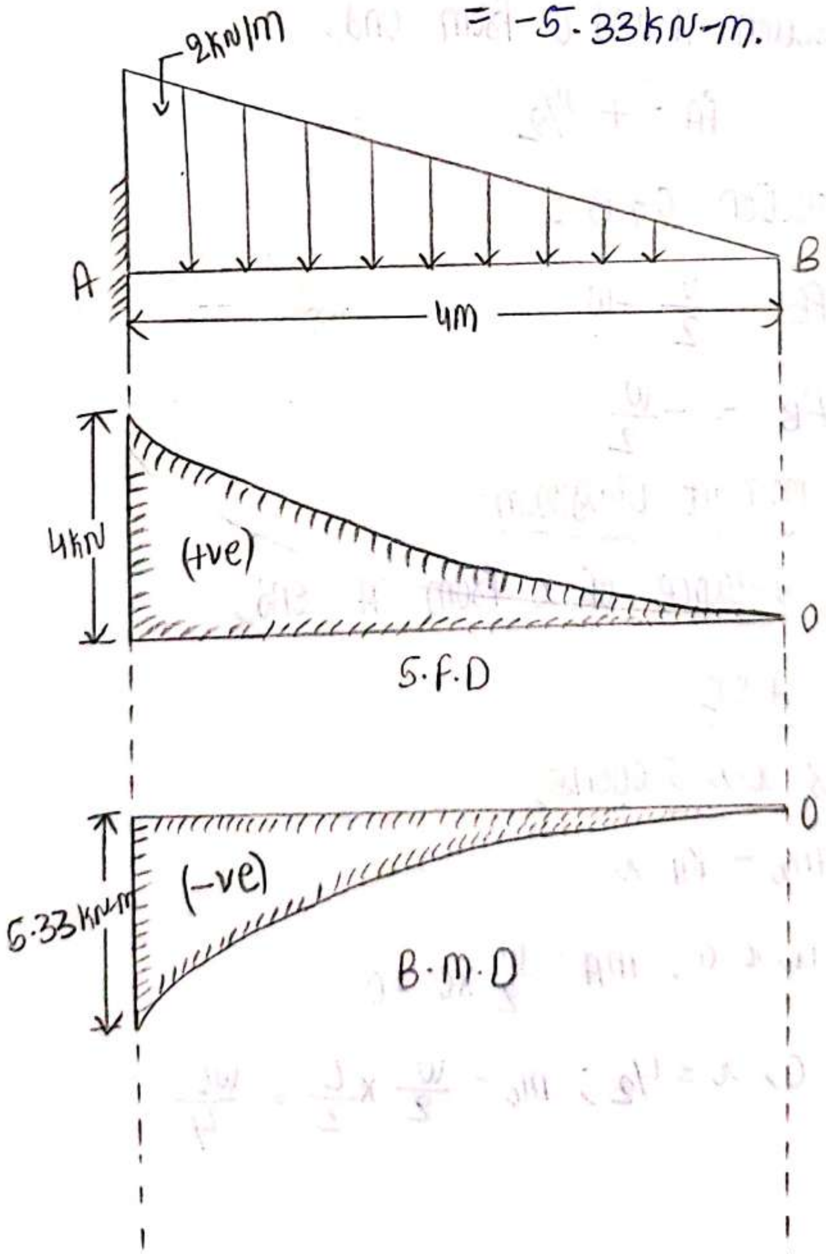
Bending moment:

At free end B; $M_B = 0$

At fixed end A; $M_A = \frac{-wL^2}{6}$

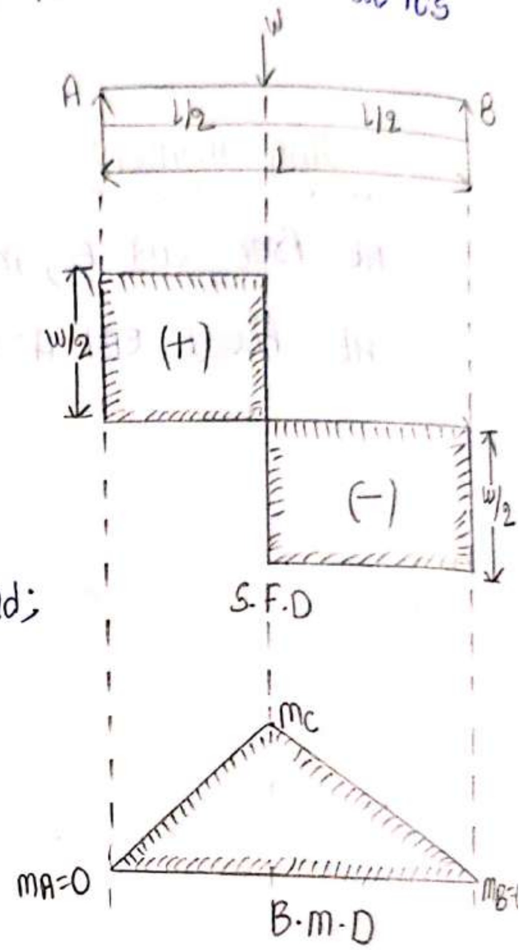
$= \frac{-2\times4^2}{6}$

$= -5.33 kN\text{-}m.$



Shear force Diagram and Bending moment Diagram for Simply Supported Beam carrying a Point Load:

A Beam AB of length "L" simply supported at the ends "A" and "B" and carrying a point load "W" at it's middle point "C".



Reaction:

Reaction support at point "A"

$$R_A = W/2$$

$$R_B = W/2$$

Shear force Diagram

At section A and C from end;

$$F_A = +W/2$$

At section C & B:

$$F_B = \frac{W}{2} - W$$

$$F_B = -\frac{W}{2}$$

Bending moment Diagram:

At a distance of x from A end,

Portion A & C:

consider x-x sections;

$$M_x = R_A \cdot x$$

At A, $x=0$; $M_A = \frac{W}{2} \times 0 = 0$

At C, $x = L/2$; $M_C = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$

Portion C & B:

consider x-x section:

$$m_x = R_A \cdot x - w\left(x - L/2\right)$$

$$= \frac{w}{2} \times x - wx + \frac{wL}{2}$$

$$= -\frac{wx}{2} + \frac{wL}{2}$$

At C, $x = L/2$

$$m_C = \frac{wL}{2} - \frac{w(L/2)}{2}$$

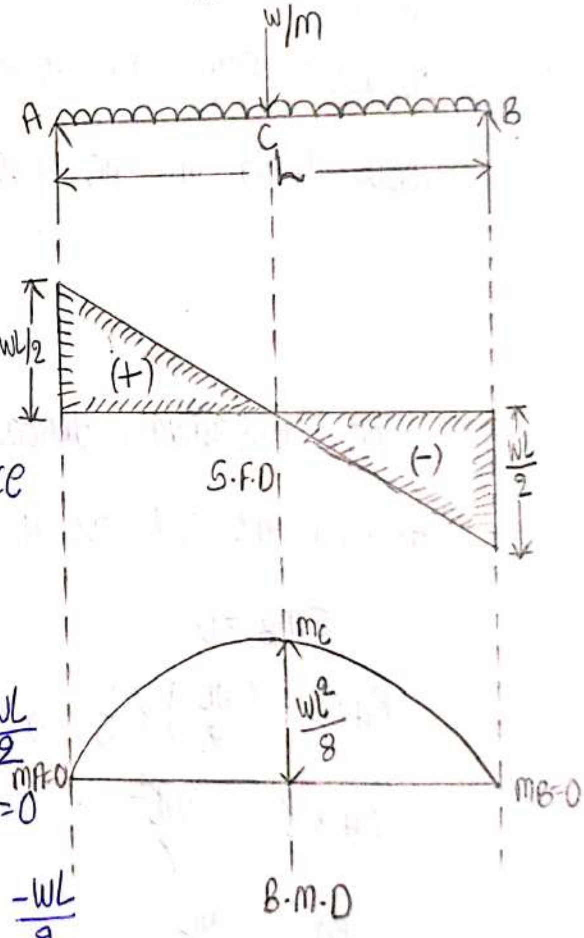
$$= \frac{wL}{2} - \frac{wL}{4}$$

$$= \frac{wL}{4}$$

At B; $x = L$

$$m_B = \frac{wL}{2} - \frac{wL}{2} = 0$$

Shear force Diagram and Bending moment Diagram for simply supported Beam carrying a (UDL) uniform Distributed load



The reaction at the supports:

At end A, $R_A = +\frac{wL}{2}$

$$R_B = +\frac{wL}{2}$$

Shear force Diagram:

consider x-x section @ a distance of "x" from end A.

$$F_x = +R_A - wx$$

At A; $x = 0$; $F_A = \frac{wL}{2} - w \times 0 = \frac{wL}{2}$

At C; $x = L/2$; $F_C = wL/2 - wL/2 = 0$

At B; $x = L$; $F_B = \frac{wL}{2} - wL = \frac{-wL}{2}$

Bending moment Diagram:

The Bending moment at a x-x distance of x from end A;

$$m_x = +R_A \cdot x - (wx)\frac{x}{2}$$

$$m_x = \frac{wL}{2} \cdot x - \frac{wx^2}{2}$$

At A, $x=0$; $M_A = \frac{wL}{2} \times 0 - \frac{w(0)^2}{2} = 0$

At C, $x = L/2$; $M_C = \frac{wL}{2} \times \frac{L}{2} - \frac{w(L/2)^2}{2} = \frac{wL^2}{4} - \frac{wL^2}{8}$

$$= \frac{2wL^2 - wL^2}{8}$$

$$= \frac{wL^2}{8}$$

At B, $x = L$, $M_B = \frac{wL}{2} \cdot L - \frac{wL^2}{2} = 0$

Shear force Diagram and Bending moment Diagram for simply supported Beam carrying a (UVL) uniformly varying load

Calculate Reactions at the supports:

Total load on the Beam = Area of the triangle ABC

$$= \frac{1}{2} \times L \times w$$

$$= \frac{wL}{2}$$

The centroidal distance from Base $= \frac{L}{3}$

Taking moment about "B"

$\Sigma M_B = 0$

$R_A \times L - \left(\frac{wL}{2}\right) \times \frac{L}{3} = 0$

$R_A \times \cancel{L} = \frac{wL^{\cancel{2}}}{6}$

$R_A = \frac{wL}{6}$

* In SSB, shear force is "zero" & B.M is maximum
* In SSB, at supports BM is

Sum of forces acting on the Beam:

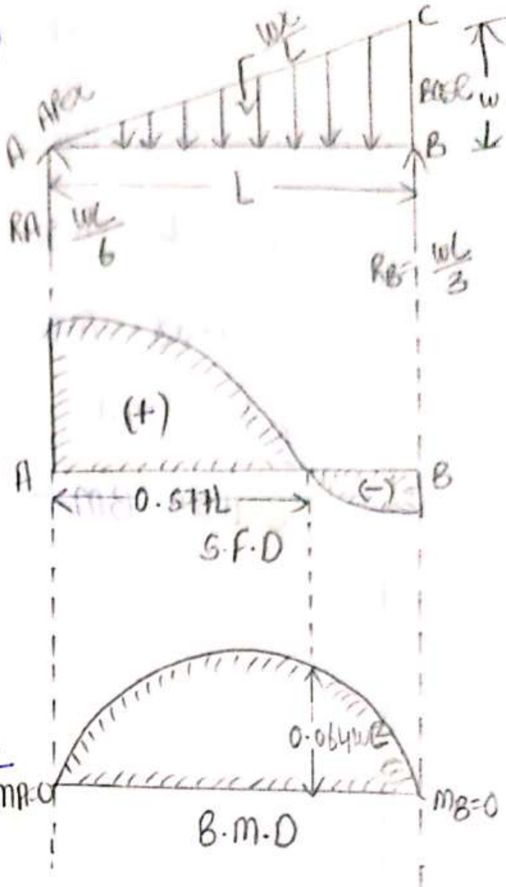
$R_A + R_B = \frac{wL}{2}$

$\frac{wL}{6} + R_B = \frac{wL}{2}$

$R_B = \frac{wL}{2} - \frac{wL}{6} = \frac{6wL - 2wL}{12} = \frac{\cancel{4}wL}{\cancel{12}_{3}} = \frac{wL}{3}$

Shear force Diagram

consider x-x section at a distance of "x" from end "A"



Total load at x-x section

$$= \frac{1}{2} \times x \times \frac{wx}{L} = \frac{wx^2}{2L}$$

shear force at x-x section is given by

$$F_x = R_A - \frac{wx^2}{2L}$$

$$F_x = \frac{wL}{6} - \frac{wx^2}{2L}$$

At A, $x=0$; $F_A = \frac{wL}{6} - \frac{w\times 0^2}{2w} = \frac{wL}{6}$

At B, $x=L$; $F_B = \frac{wL}{6} - \frac{wL^{\cancel{2}}}{2\cancel{L}}$

$$= \frac{2wL - 6wL}{12}$$

$$= \frac{-4wL}{12} = \frac{-wL}{3}$$

The shear force will be zero i.e,

$$\frac{wL}{6} - \frac{wx^2}{2L} = 0$$

$$\frac{\cancel{w}L}{6} = \frac{\cancel{w}x^2}{2L}$$

$$\frac{\cancel{2}L^2}{\cancel{6}\,3} = x^2$$

$$x = \sqrt{\frac{L^3}{3}} = \frac{L}{\sqrt{3}} = 0.577L$$

Bending moment Diagram

consider x-x section at a distance of "x" from end A.

Bending moment at x-x section is given by

$$M_x = R_A x - \frac{wx^2}{2L} \times \frac{x}{3}$$

$= \frac{wL}{6} x - \frac{wx^3}{6L}$

At A, x=0 ; $m_A = \frac{wL(0)}{6} - 0 = 0$

At B, x=L ; $m_B = \frac{wL^2}{6} - \frac{wL^2}{6} = 0$

The shear force is zero at $x = 0.577L$ where Bm will be max.

$$\text{max. Bm} = \frac{wLx}{6} - \frac{wx^3}{6L}$$

$$= \frac{wL \times 0.577L}{6} - \frac{w \times 0.577^3 \times L^3}{6L}$$

$$= 0.096wL^2 - 0.032wL^2$$

$$= (0.096 - 0.032)wL^2$$

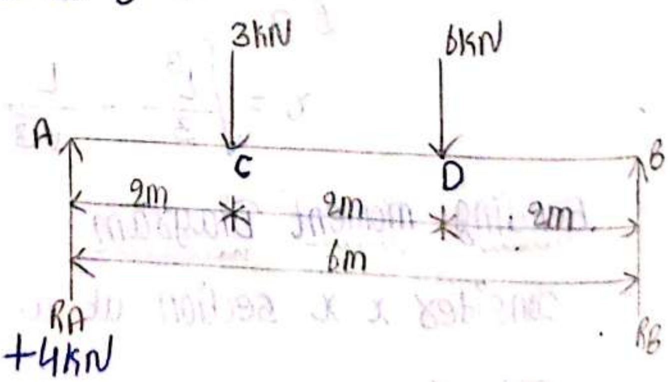
$$m = 0.064 wL^2$$

1) A simply supported beam of length 6m carries a point load of 3kN and 6kN at a distance of 2m and 4m from the left end. Draw S.F.D and B.M.D.

Sol) Given data

1) Calculate Reactions at the supports:

Taking moments about B



$\Sigma M_B = 0$

$R_A \times 6 - 3 \times 4 - 6 \times 2 = 0$

$R_A \times 6 - 24 = 0$

$R_A = 24/6 = +4kN$

Sum of forces acting on Beam

$R_A + R_B = 3 + 6$

$4 + R_B = 9 \Rightarrow R_B = 9 - 4 = 5kN$

ii) Shear force Diagram:

SF at A; $F_A = +R_A = +4kN$

SF at b/w A & C will be constant $= 4kN$

SF at C; $F_C = R_A - 3 = 4-3 = +1kN$

SF b/w C & D will be constant $= 1kN$

SF at D: $F_D = R_A - 3 - 6 = 4-3-6 = -5kN$

SF b/w D & D will be constant $= -5kN$

iii) Bending moment Diagram

BM at A, $M_A = 0$

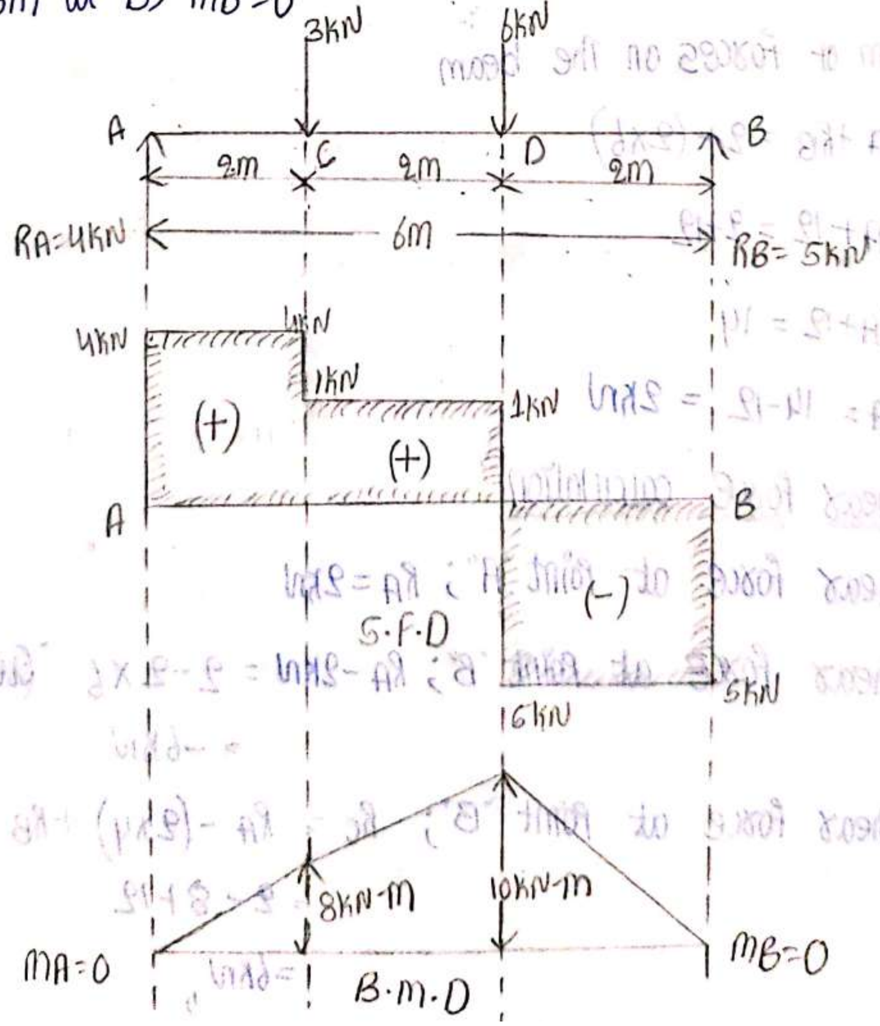
BM at C, $M_C = R_A \times 2 = 4 \times 2 = 8kN\text{-}m$

BM at D, $M_D = R_A \times 4 - 3 \times 2$

$= 4 \times 4 - 3 \times 2$

$= 10kN\text{-}m$

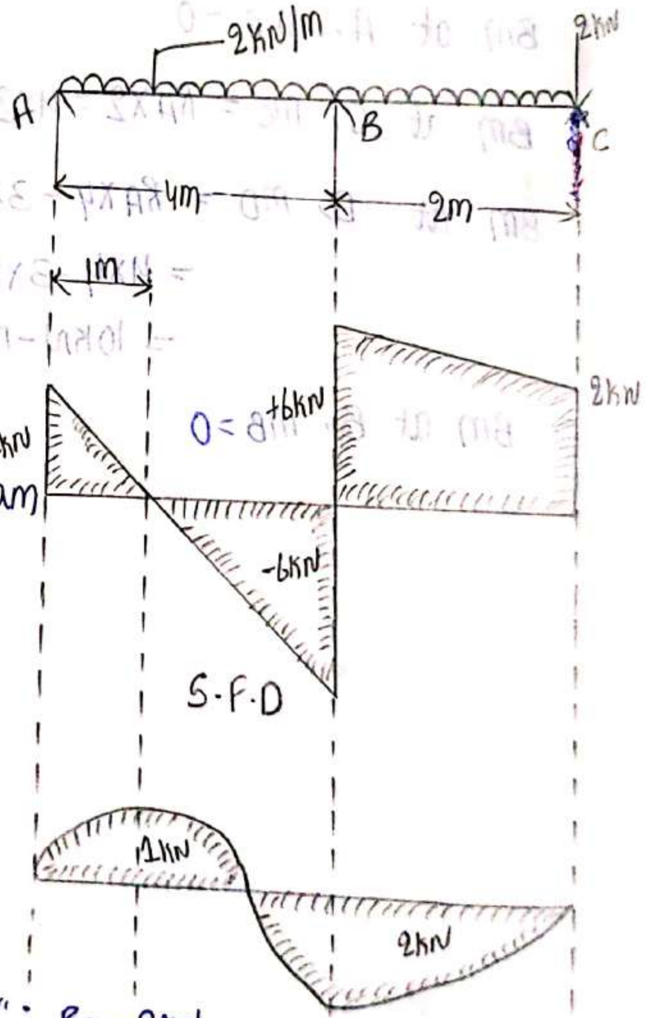
BM at B, $M_B = 0$



over hanging beams 21/08/2024

In over hanging beam the Bending moment is positive between the two supports the bending moment is negative for the over hanging portion, at some point the bending moment is zero after changing its sign from positive to negative That point is known as "point of contra flexure"

1) Draw the S.F and B.M diagram for given over hanging beam.



Taking moment about "A"

$R_B \times 4 = 2\times6\times\frac{6}{2} + (2\times6)$

$R_B\times4 = 36+12$

$R_B = 12kN$

Sum of forces on the beam

$R_A + R_B = 2 + (2\times6)$

$R_A + 12 = 2+12$

$R_A + 12 = 14$

$R_A = 14-12 = 2kN$

shear force calculation

shear force at point "A"; $R_A = 2kN$

B.m.D

shear force at point "B"; $R_A - 2kN = 2 - 2\times6$ "(left)"

$= -6kN$

shear force at point "B"; $R_C = R_A - (2\times4) + R_B$ "(Right)"

$= 2-8+12$

shear force $f_C = R_A - (2\times4) + R_B - (2\times2)$

$= 2-8+12-4$

$= 2kN$

shear force at a portion AB at a distance x from A

$f(x) = R_A - 2x$

$0 = R_A - 2x$

$2-2x=0$

$x = 1m$

Bending moment diagram

B.M at A ; $M_A = 0$

B.M at B; $M_B = R_A \times 4 - 2\times4\times\frac{4}{2}$

$= 2\times4 - 16 = -8kN\text{-}m$

B.M at C, $M_C = R_A \times 16 - 2\times6\times\frac{6}{2} + R_B \times 2$

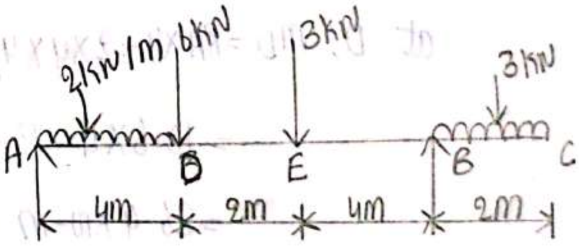
$= 2\times6 - 36 + 12\times2$

2) Draw the S.F and B.M for over hanging Beam

Sol) Given data

calculate reactions

Taking moment about A



$R_B \times 10 = 2\times4\times\frac{4}{2} + 6\times4 + (3\times6) + (3\times2)\times\left(\frac{2}{2}+10\right)$

$R_B \times 10 = 16+24+18+66$

$R_B = 12.4kN$

sum of forces on the beam

$R_A + R_B = (2\times4)+6+3+(3\times2)$

$R_A + 12.4 = 8+6+3+6$

$R_A = 10.6kN$

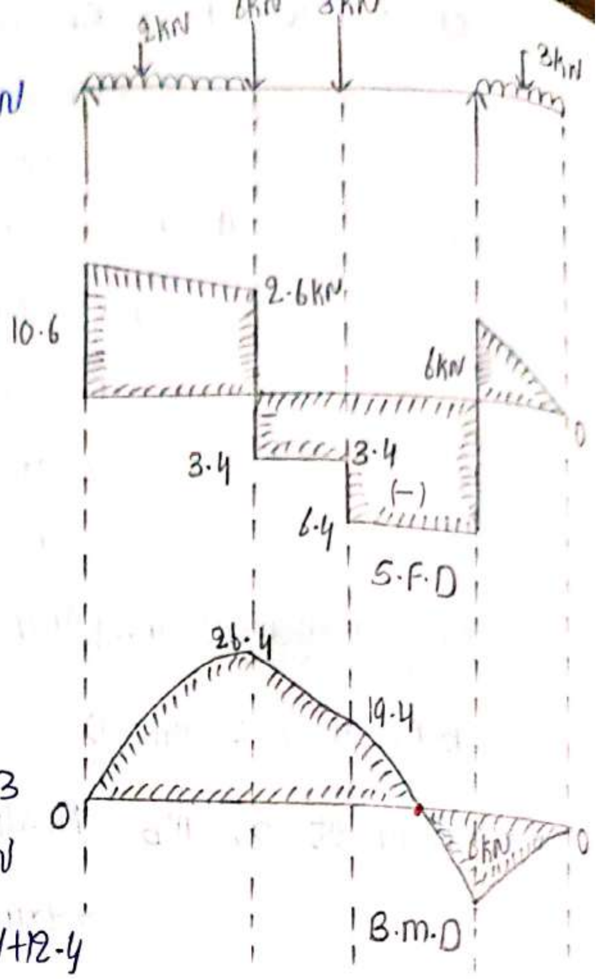
Shear force Diagram

S.F at A; $F_A = R_A = +10.6 kN$

S.F at D; $F_D = R_A - 2\times4$

$= 10.6 - 8$

$= 2.6 kN$



Shear force at E, F_E

$= R_A - 2\times4 - 6$

$= 10.6 - 8 - 6$

$= -3.4 kN$

S.F at B; (left) $F_B = -3.4 - 3$

$= -6.4 kN$

S.F at B; (Right) $F_B = -6.4 + 12.4$

$= 6 kN$

Bending moment Diagram

B.m at A; $m_A = 0$

at D; $m_D = R_A \times 4 - 2\times4\times4/2$

$= 10.6\times4 - 16$

$= 26.4 kN\text{-}m$

B.m at "E". $m_E = R_A\times6 - 2\times4\times(4/2+2) - 6\times2$

$= 10.6\times6 - 32 - 12 = 19.6 kN\text{-}m$

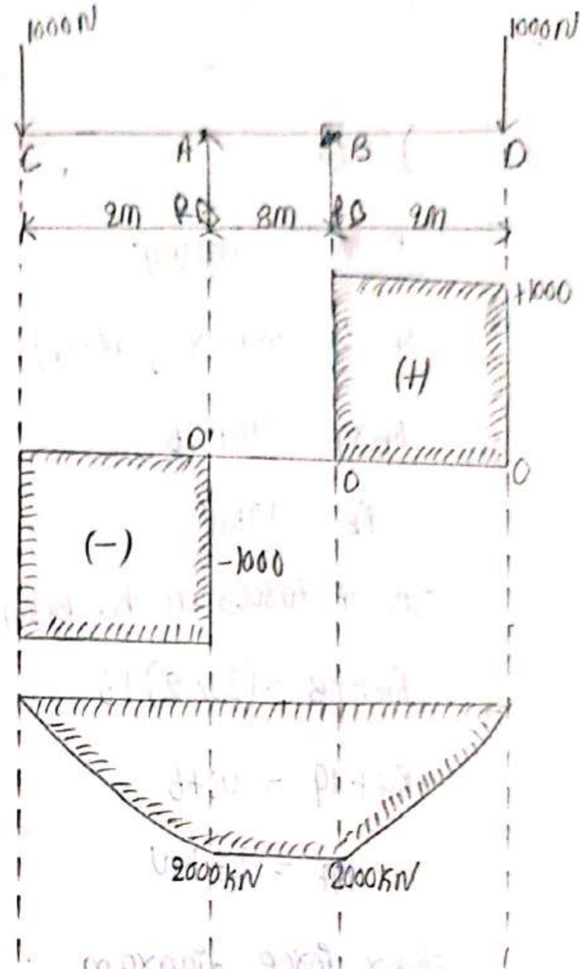
B.m at "B". $m_B = R_A\times10 - 2\times4\times(\frac{\cancel{4}^{2}}{\cancel{2}_{1}}\times2+4) - 6\times6 - 3\times4$

$= 10.6\times10 - 64 - 36 - 12 = -6 kN\text{-}m$

B.m at "C" $m_C = 10.6\times12 - 2\times(\frac{4}{2}+2+4+2) - 6\times8 - 3\times6\times$

$12.4\times$

3) calculate reactions taking moment A:



$R_B \times 8 = -1000 \times 2 + 1000 \times 10$

$R_B \times 8 = -2000 + 10{,}000$

$R_B = \frac{8000}{8} = 1000\,kN$

sum of forces on the beam

$R_A + R_B = 1000 + 1000$

$R_A = 2000 - 1000$

$= 1000\,kN$

shear force Diagram

S.F at "C" $= R_A = 1000$

S.F at A $= R_A - 1000$

$= +1000 - 1000$

$= 0$

shear force at "D" $= +R_A - 1000 + 1000$ (Right)

$= \cancel{0}+1000 - \cancel{1000} + \cancel{1000}$

$= 1000\,kN$

shear force at "D" $= R_A - 1000 + 1000 - 1000$

$= 1000\,kN$

Bending moment Diagram

B.M At C; $M_C = 0$

B.M at A; $M_A = -2 \times 10000 = -2000\,kN\text{-}m$

B.M at B; $M_B = -1000 \times 10 + 1000 \times 8$

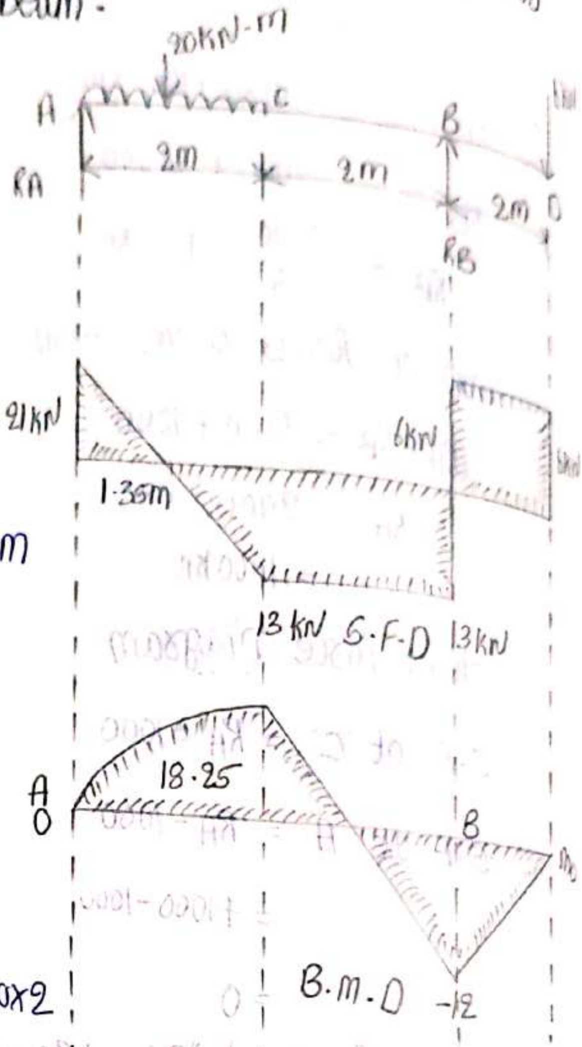
$= -10{,}000 + 8000$

$= -2000\,kN\text{-}m$

At 0; $M_0 = 0$

4) Draw the shear force and Bending moment diagram for one side over hanging beam.

Sol) Given data.



Calculating reaction

$R_B \times 4 = 20 \times 2 \times \frac{2}{2} + (6 \times 6)$

$R_B \times 4 = 40 + 36$

$R_B = 19kN$

Sum of forces on the Beam

$R_A + R_B = (20 \times 2) + 6$

$R_A + 19 = 40 + 6$

$R_A = 27 kN$

Shear force diagram

S.F at C; $F_C \cdot R_C = R_A - 20 \times 2$

$= 27 - 40 = 13kN$

There is no load b/w C&B Portion

At B, F_B (right) $= R_A - 20 \times 2 + R_B$

$= -13 + 19$

$= 6 kN$

At C; $F_D = 6 - 6 = 0$

at a Portion AC at a distance x from A

$f(x) = R_A - 20x$

$0 = R_A - 20x$

$27 - 20x = 0$

$x = \frac{\overset{13.5}{\cancel{27}}}{20} = 1.35m$

Bending moment diagram

B.m at $A = 0$

B.m at $C = R_A \times 2 - 20 \times 2 \times \frac{2}{2}$

$= 27 \times 2 - 40$

$= 14 kN-m$

B.m at $m_B = R_A \times 4 - 20 \times 2 \left(\frac{2}{2} + 2\right)$

$= 27 \times 4 - 40 \times 3$

$= -12 kN-m$

At Point "D" $= R_A \times 6 - 20 \times 2 \times \frac{2}{2} + 4) + R_B \times 2$

$= 27 \times 6 - 40 \times 5 + 19 \times 2$

$= 0$

maximum B.m at a distance.

1-35m

$m_{max} = R_A \times 1.35 - 20 \times 1.35 \times \frac{1.35}{2}$

$= 27 \times 7.35 - 20 \times \frac{1.35^2}{2}$

$= 18.225 kN-m$

$m_x = R_A \times x - 20 \times x \left(\frac{x}{2} + 2\right)$

$0 = 27 \times x - 20 \times x \left(\frac{x}{2} + 2\right)$

$0 = 27 \times x - \frac{20x^2}{2} - 40x$

$0 = 27x - 10x^2 - 40x$

$-10x^2 = -13x$

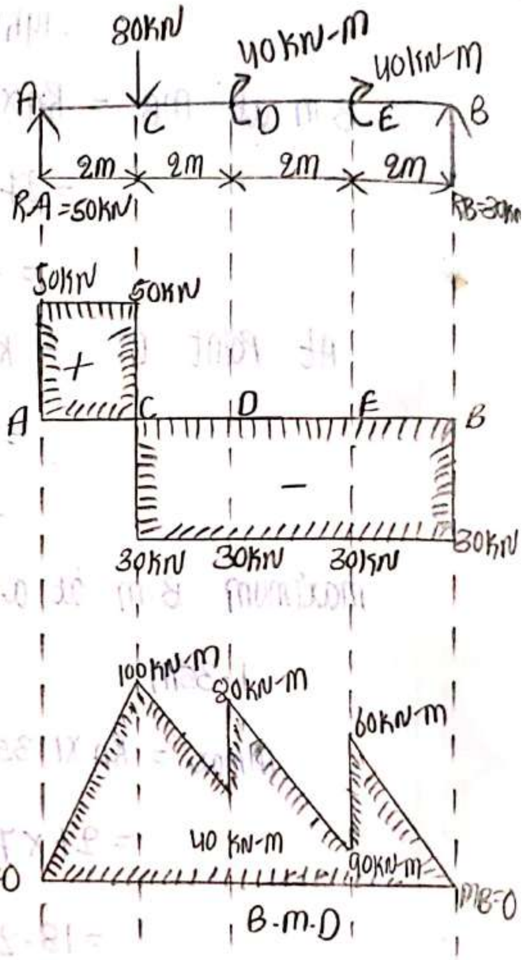
$\boxed{x = \frac{13}{10} = 1.3m}$ → Point of contra flexure

22/08/2024

couple moment

1) A simply supported beam AB of length 8m as shown in figure calculate the shear force and bending moment values at different points.

Sol) calculate reactions at supports



Taking moment about 'A'

$R_B \times 8 = 80\times 2 + 40 + 40$

$R_B \times 8 = 160 + 80 = 30kN$

sum of the forces acting on the beam

$R_A + R_B = 80$

$R_A + 30 = 80$

$R_A = 50\,kN$

i) shear force diagram

S.F at A, $F_A = R_A = 50kN$

S.F at C, $F_C = R_A = 50kN$

S.F at D; $F_D = R_A - 80 = -30kN$

S.F at E, $F_E = R_A - 80 = -30kN$

S.F at B, $F_B = R_A - 80 = -30kN$

ii) Bending moment diagram

B.M at A, $M_A = 0$

B.M at C, $M_C = R_A \times 2$

$= 50\times 2 = 100\,kN-m$

B.M at D, M_D (left) $= R_A \times 4 - 80\times 2 = 40kN-m$

at D, M_D (right) $= R_A \times 4 - 80\times 2 + 40 = 80kN-m$

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B-m at E, m_E (left) $= R_A \times 6 - 80 \times 4 + 40 = 20$ kN-m

m_E (right) $= R_A \times 6 - 80 \times 4 + 40 + 40 = 60$ kN-m

B-m at B, $m_B = R_A \times 8 - 80 \times 6 + 40 + 40 = 0$

2) Draw the S.F.D and B.M.D

sol) calculate reactions at supports

Taking moment at "B"

$$R_A \times 5 = 4 \times 2 \times \left(\frac{2}{3} + 3\right) - 9 + 12$$

$$R_A = 7 \text{kN}$$

sum of forces acting on the beam.

$$R_A + R_B = 8 + 6 - 7$$

$$R_B = 7 \text{kN}$$

1) Shear force Diagram

S.F at A, $F_A = 7$ kN

S.F at C, $F_C = R_A - 4 \times 2 = 1$ kN

There is no load from C & E

i.e S.F.D is constant 1kN

S.F at E, $F_C = R_A - 4 \times 2$

$= 7 - 8 = 1$ kN

S.F at B, $F_B = R_A - 4 \times 2 - 6$

$= 7 - 8 - 6 = -7$ kN

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3) Draw the shear force Diagram and Bending moment Diagram for a simply supported beam with couple moment at the ends.

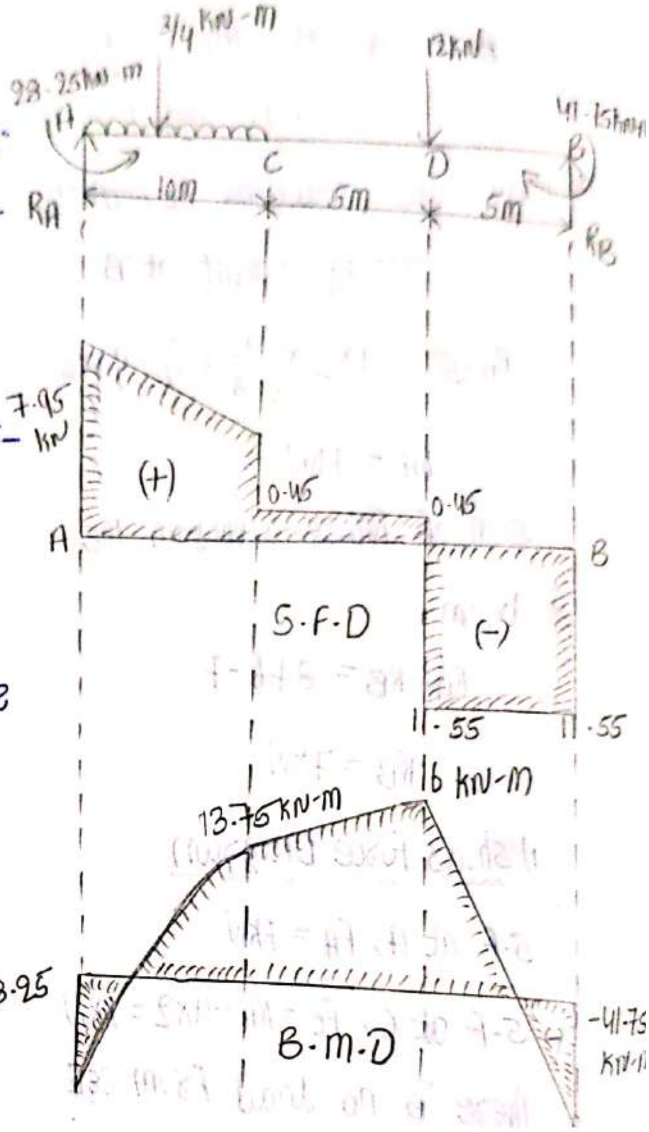
Sol) Calculate Reactions about B.

$R_A \times 20 = \frac{3}{4} \times 10 \times \left(\frac{10}{2} + 10\right) + 12 \times 5$

$= +28.25 - 41.75$

$R_A \times 20 = \frac{3}{4} \times 10 \times 15 + 60 + 28.25 - 41.75$

$\boxed{R_A = 7.95\,kN}$



Sum of forces acting on the beam,

$R_A + R_B = 7.5 + 12$

$7.95 + R_B = 7.5 + 12$

$\boxed{R_B = 11.55\,kN}$

Shear force Diagram

S.F at A; $F_A = +R_A = 7.95\,kN$

S.F at C; $F_C = R_A - \frac{3}{4} \times 10$

$= 7.95 - 7.5$

$= 0.45\,kN$

S.F at D; $F_D = R_A - \frac{3}{4} \times 10 = 0.45\,kN$

S.F at B; $F_B = R_A - \left(\frac{3}{4} \times 10\right) - 12$

$= 7.95 - 7.5 - 12$

$= -11.55\,kN$

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Bending Moment Diagram

B.M at A ; $M_A = -28.25$ kN-m

B.M at C ; $M_C = R_A \times 10 - \left(\frac{3}{4} \times 10 \times \frac{10}{2}\right) - 28.25$

$= 7.95 \times 10 - 7.5 \times 5 - 28.25 = 13.75$ kN-m

B.M at D ; $M_D = R_A \times 15 - \frac{3}{4} \times 10 \times \left(\frac{10}{2} + 5\right) - 28.25$

$= 7.95 \times 15 - 7.5 \times 10 - 28.25$

$= 16$ kN-m

B.M at B ; $M_B = 7.95 \times 20 - \frac{3}{4} \times 10 \times \left(\frac{10}{2} + 10\right) - 28.25 - (12 \times 5)$

$= 7.95 \times 20 - 7.5 \times 15 - 28.25 - 60$

$= -41.75$ kN-m

4) Draw SFD & BMD for double overhanging beam.

Sol) Calculate Reactions at supports

Taking moments about A

$R_B \times 8 = -1000 \times 2 + 1000 \times 10$

$R_B \times 8 = -2000 + 10000$

$R_B = \frac{8000}{8} = 1000$ kN

Sum of forces acting on the beam

$R_A + R_B = 1000 + 1000$

$R_A + 1000 = 2000$

$R_A = 2000 - 1000$

$= 1000$ kN

i) Shear force Diagram :

SF at C; $F_C = -1000\,kN$

SF at A; $F_A = R_A - 1000 = 1000 - 1000 = 0$

SF at D; $F_D = \overset{-1000 + R_A + R_B}{R_A - 1000 + 1000}$ (left)

$= 1000 - \cancel{1000} + \cancel{1000}$

$= 1000\,kN$

SF at D; $F_D = R_A - 1000 + R_B - 1000$

$= 1000 - 1000 + 1000 - 1000$

$= 1000\,kN$

ii) Bending moment Diagram :

BM at C; $M_C = 0$

BM at A; $M_A = -1000 \times 2$

$= -2000\,kN\text{-}m$

BM at B; $M_B = -1000 \times 10 + R_A \times 8$

$= -10000 + 8000$

$= -2000\,kN - m$

BM at D; $M_D = -1000 \times 12 + R_A \times 10 + R_B \times 2$

$= -12000 + 10000 + 2000$

$= 0$

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Unit-3 28/08/2024

Flexural and shear stress

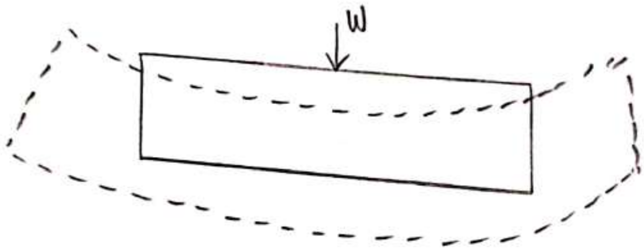
I) Flexural stresses

Bending stress (σ_b (or) σ_f)

Bending stresses are the internal resistance developed due to external forces which causes bending of a member.

⇒ It is the normal stress i.e; induced at a point in the body subjected to load that causes it to bend.

⇒ It is also called as flexural stress (σ_f).



Theory of simple Bending:

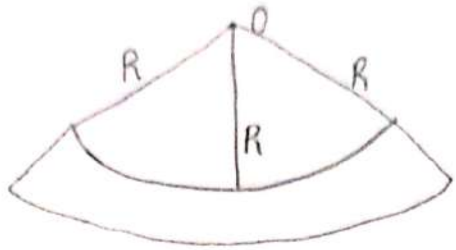
Assumptions:

1) The material of the Beam is homogeneous and isotropic.

2) The value of youngs modulus will be same in tension and compression zone.

3) The traverse section (or) cross-section which were plane before bending remain plane after bending.

4) The beam is initially straight and all longitudinal filaments or elements bend into circular arc with a common centre of curvature.

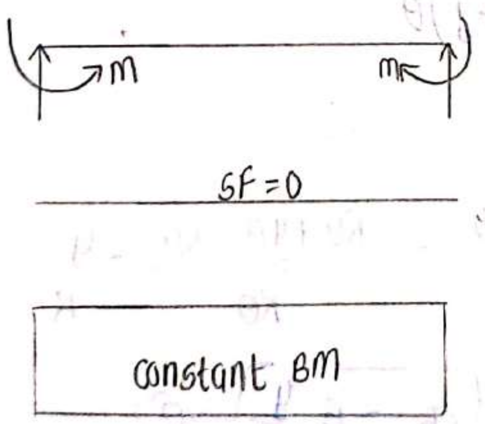


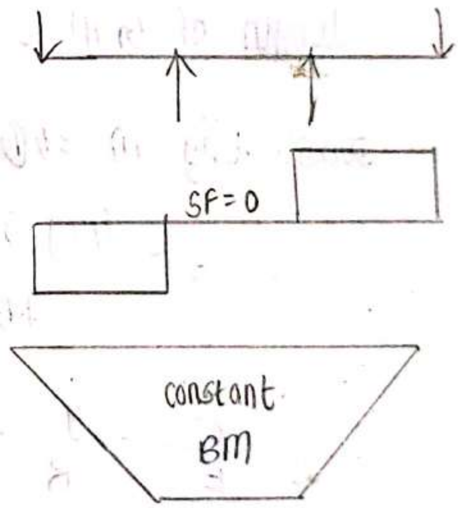
5) The radious of curvature (R) is large compared to the dimension of the cross-section.

6) Each layer of the beam is free to expand.

Pure bending

If a length of the beam is subjected to a constant Bending moment and no shear force then the stresses will be setup in the beam due to bending moment only. That beam is said to be pure bending

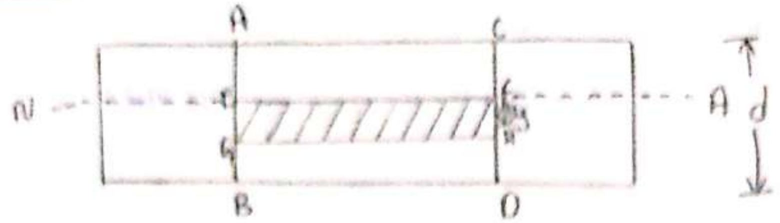




Bending equation

1) Consider (S-S-B) simply supported Beam with span length L

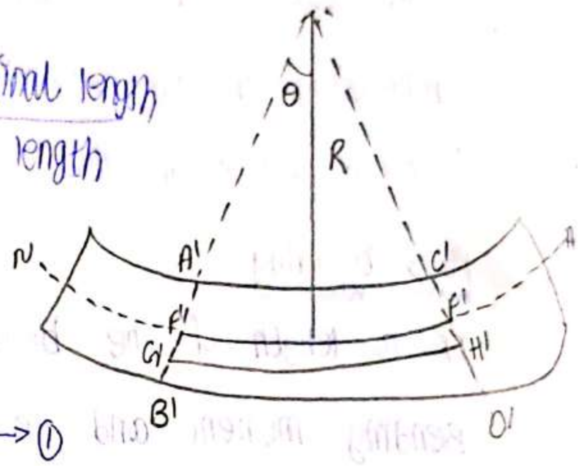
2) Consider the beam section before bending consider another layer 'GH' at a distance 'y' from neutral axis.



3) consider the section of the beam after bending strain in layer GH due to bending.

4) strain $e = \frac{\text{change in length}}{\text{original length}}$

$= \frac{\text{Final length - original length}}{\text{original length}}$



$= \frac{G'H' - GH}{GH}$

$e = \frac{G'H' - EF}{EF} \rightarrow (1)$

Length of arc, $EF = GH = R \times \theta$

Length of $G'H' = (R+y)\theta$

substituting in eq (1)

$e = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$

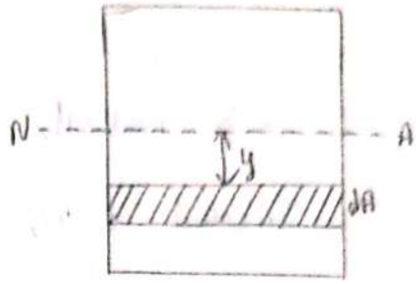
$\frac{\sigma}{E} = \frac{y}{R} \Rightarrow \boxed{\sigma_b = E\frac{y}{R}} \rightarrow (2)$

$\sigma_b = \frac{E}{R} y$

$\frac{\sigma_b}{y} = \frac{E}{R} \rightarrow (3)$

5) Bending stress $\sigma_b \propto y$

consider the c/s of beam as rectangle and consider the element strip having area dA located at a distance of y from NA.



force acting on the small element

$$\sigma = \frac{F}{A} \Rightarrow dF = \sigma_b \times dA$$

Bending moment on the element at a distance of y from NA is

$$dm = dF \times y$$

$$dm = \sigma_b \times dA \times y$$

$$dm = \frac{E}{R} y \times dA \times y$$

$$dm = \frac{E}{R} y^2 dA$$

Total moment on the Beam

By applying integration

$$\int dm = \int \frac{E}{R} x y^2 \, dA$$

$$m = \frac{E}{R} \int y^2 dA$$

$$m = \frac{E}{R} I \quad \text{[Based on second moment area method]}$$

$$\frac{m}{I} = \frac{E}{R} \rightarrow (4)$$

equating (3) & (4)

$$\frac{E}{R} = \frac{m}{I} = \frac{f/\sigma_b}{y}$$

$$\boxed{\frac{m}{I} = \frac{\sigma_b F}{y} = \frac{E}{R}}$$

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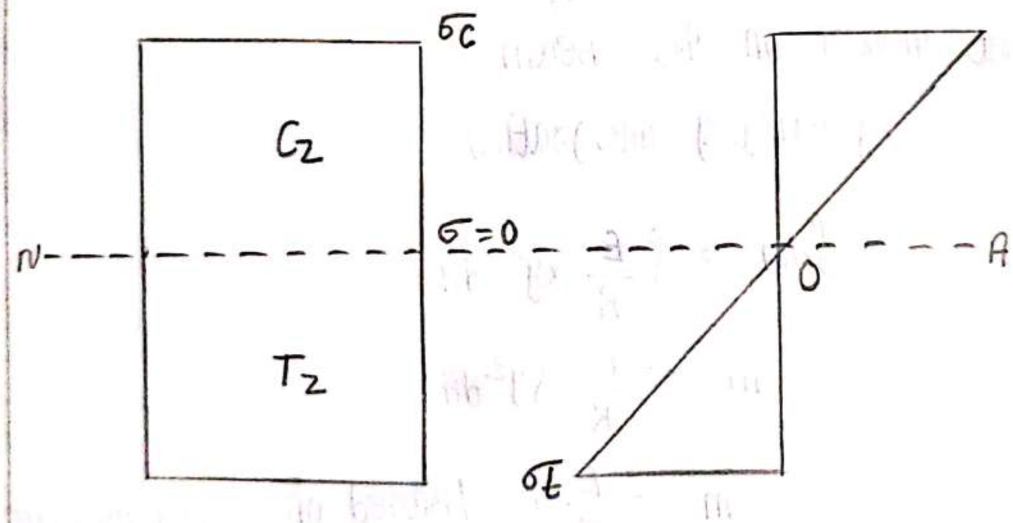
Neutral axis

it is a line of intersection of neutral plane (or) neutral layer of a cross section is called neutral axis of the section.

⇒ the neutral axis of the beam passes through the centroid of the section.

⇒ on one side of the neutral axis is compressive stress (or) compressive zone and other side is tension zone.

⇒ at the neutral axis bending stress or bending strain is zero.



Section modulus ('Z')

it is defined has the ratio of moment of inertia of a section about the neutral axis to the Distance from the N.A to outer most layer

$$Z = I/y$$

from Bending equation

$$\frac{m}{I} = \frac{\sigma_b}{y}$$

$$m = \sigma_b \times \frac{I}{y} \Rightarrow \boxed{m = \sigma_b \times Z}$$

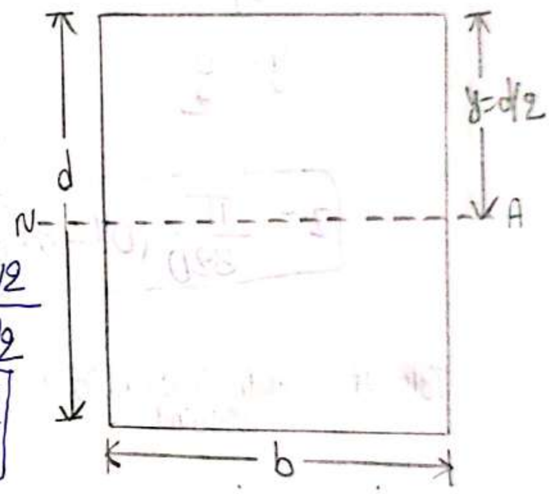
thus stress "σ" will be maximum when "y" is maximum hence at that point the section modulus "Z" will be maximum.

Section modulus for various shape of Beam sections.

1) Rectangular section

moment of inertia at Rectangular section

$$I = \frac{bd^3}{12}$$



Distance from N.A to outer most layer $= y = d/2$

Section modulus $= I/y = \frac{bd^3/12}{d/2}$

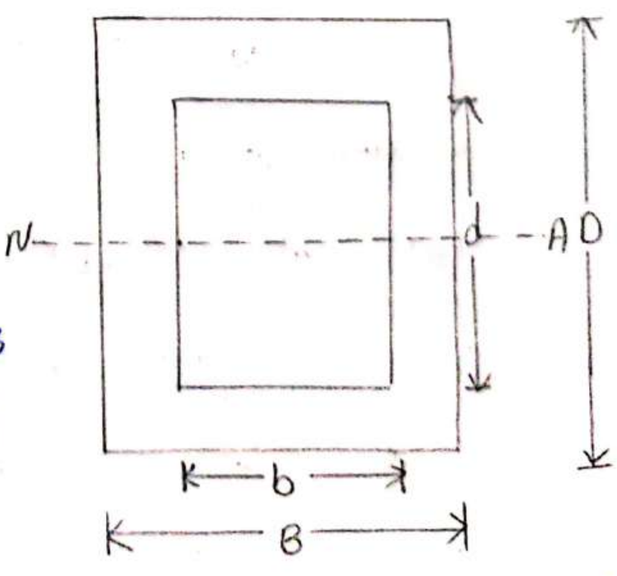
$$\boxed{Z = \frac{bd^2}{6}}$$

2) Hollow Rectangular section

moment of inertia at hollow Rectangular section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y = D/2$$



Section modulus

$$Z = I/y = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{D/2}$$

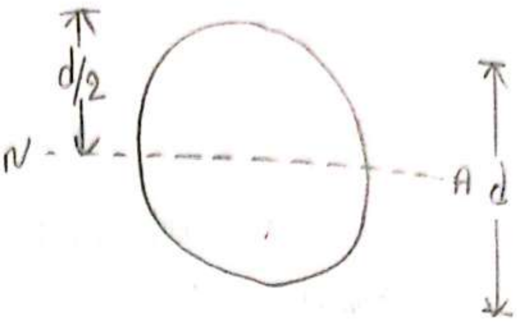
$Z =$

3) Circular section

Moment of inertia at circular section

$$I = \frac{\pi}{64} d^4$$

$$y = D/2$$



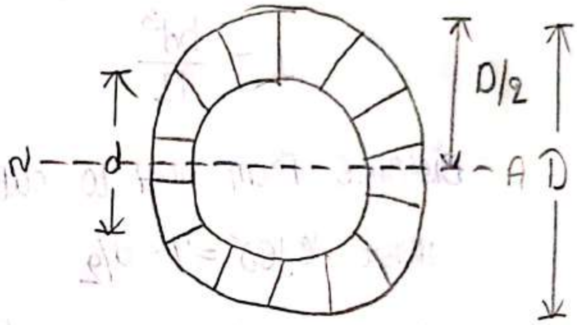
$$Z = \frac{\frac{\pi}{64} d^4}{D/2} = \frac{\frac{\pi}{32} d^3}{D}$$

4) Hollow circular section

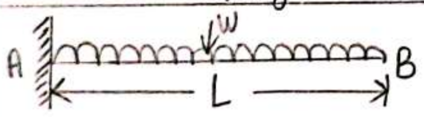
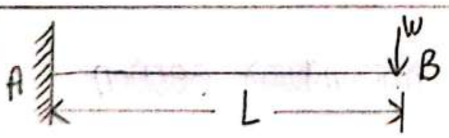
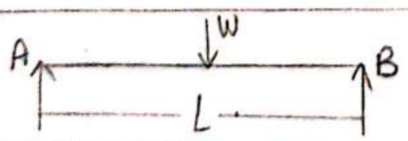
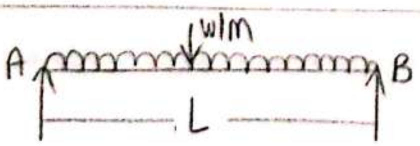
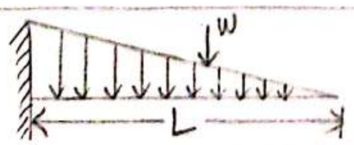
Moment of inertia at Hollow circular section

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$y = \frac{D}{2}$$

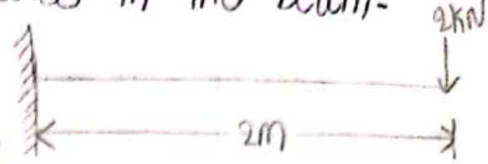


$$\boxed{Z = \frac{\pi}{32D}(D^4 - d^4)}$$

Type of Beam subjected to loading	Shear force (F)	Bending moment
	$F = wL$	$m = \frac{-wL^2}{2}$
	$F = W$	$m = WL$
	$F = \frac{W}{2}$	$m = \frac{WL}{4}$
	$F = \frac{wL}{2}$	$m = \frac{wL^2}{8}$
	$\frac{wx^2}{2L}$	$\frac{-wx^3}{2L}$

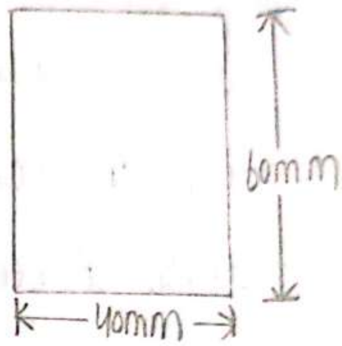
Problems

1) A cantilever of length 2m when a load of 2kN is applied at the free end. If the section of the beam is 40mm x 60mm. Find the stress in the beam.



Sol) Given data

Length of beam "L" = 2m = 2000mm



load "W" = 2kN = 2×10^3 N

width of the beam "b" = 40mm

Depth of the beam "d" = 60mm

Bending stress σ_b = ?

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$\sigma_b = \frac{M}{I}\times y$$

$$\sigma_b = \frac{M}{Z}$$

For cantilever beam, Bending moment

$$M = -WL$$
$$= -2\times2$$
$$= -4\text{kN-m}$$
$$= -4\times10^6 \text{ N-mm}$$

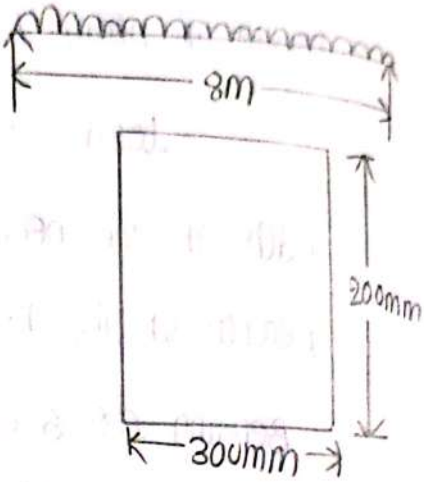
section modulus, $Z = \frac{bd^2}{6} = \frac{40\times60^2}{6} = 24\times10^3 \text{ mm}^3$

Bending stress, $\sigma_b = \frac{M}{Z} = \frac{-4\times10^6}{24\times10^3} = -166.67 \text{ N/mm}^2$

$= 166.67 \text{ N/mm}^2$ (Tensile)

2) A rectangular beam 200mm deep & 300mm wide is a simply supported beam over a span of 8m. what UDL per m the beam may carry, if the σ_b is not to exceed $120 N/mm^2$.

Sol) Given data



Span of Beam "L" = 8m = 8000mm

width of Beam "b" = 300mm

Depth of Beam "d" = 200mm

Bending stress "σ_b" = $120 N/mm^2$

For SSB, Bending moment, $m = \frac{wL^2}{8} = \frac{w \times 8000^2}{8}$

$= 8 \times 10^6 w$

Section modulus "Z" $= \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 20 \times 10^5 mm^3$

Bending stress, "σ_b" $= \frac{m}{Z}$

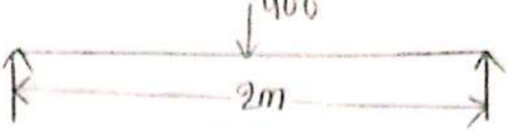
$$120 = \frac{8 \times 10^6 w}{20 \times 10^5}$$

$$w = \frac{120 \times 20 \times 10^5}{8 \times 10^6} = 30 N/m$$

3) A square beam 20mm x 20mm in section & 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of a beam. what UDL per m length will break a cantilever of the same material 40mm wide, 60mm deep & 3m long.

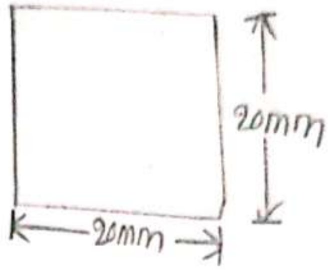
Sol) Given data

S.S.B



Breadth of Beam "b" = 20mm

Depth of Beam "d" = 20mm



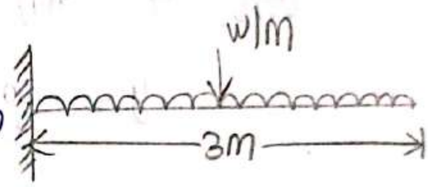
length of Beam "L" = 2m = 2000mm

load "W" = 400N

Bending moment, "m" = $\frac{WL}{4} = \frac{400 \times 2000}{4} = 2 \times 10^5$ N-mm,,

Bending stress, $\sigma_b = \frac{m}{Z} = \frac{2 \times 10^5}{20 \times 20^2/6} = 150$ N/mm^2,,

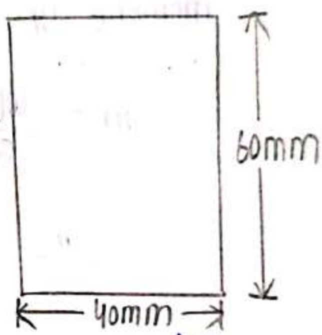
4) cantilever Beam



Length of Beam, "L" = 3m = 3000mm

width of Beam, "b" = 40mm

Depth of Beam, "d" = 60mm



Bending moment, "m" = $\frac{-wl^2}{2}$

$= \frac{-w \times 3^2}{2} = -4.5w$ Nm

section modulus, "Z" = $\frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000$ mm^3,,

Bending stress, $\sigma_b = \frac{m}{Z}$

$\frac{-w \times 3^2}{2} = -4.5w$Nm

$= -4.5w \times 1000$

$= -4500w$ mm

$150 = \frac{-4500w}{24000}$

$-4500w = 150 \times 24000$

$w = \frac{150 \times 24000}{-4500}$

$= -800$ N/mm

$w = -8 \times \cancel{10^3}^{10^2} \frac{N}{m \times \cancel{1000}}$

$= -800$ N/m

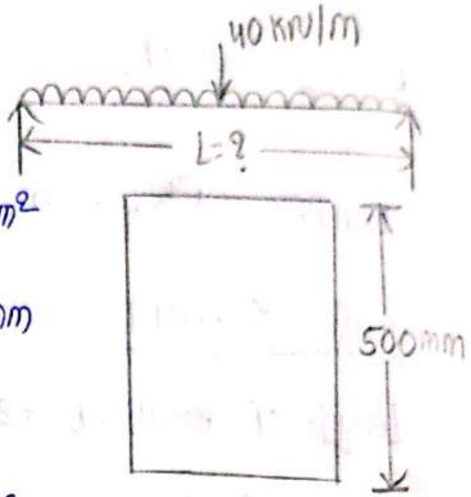
$150 = \frac{\frac{-w \times 3^2}{2}}{24000}$

$150 = \frac{-w \times 3^2}{2 \times 24000}$

$w = \frac{150 \times 2 \times 24000}{-3^2} = -8 \times 10^2$ N/m (Tensile)

5) A beam is SSB and carries a UDL of 40kN/m run over the whole span. the section of beam is rectangular having a depth of 500mm. If the maximum stress in the material of the beam is 120N/mm^2 and moment of inertia of the section is 7×10^8 mm^4. find the span of the beam.

Sol) Given data



UDL "w" = 40 kN/m

maximum stress "f" = 120 N/mm^2

Depth of beam "d" = 500mm

$$y = \frac{500}{2} = 250\text{mm}$$

moment of inertia $I = 7\times10^8$ mm^4

$$m = \frac{wL^2}{8} = \frac{40\times L^2}{8} = 5L^2 \text{ kN/m} = 5\times10^6 L^2 \text{ Nmm}$$

$$\frac{m}{I} = \frac{f}{y} \Rightarrow m = \frac{I}{y} f$$

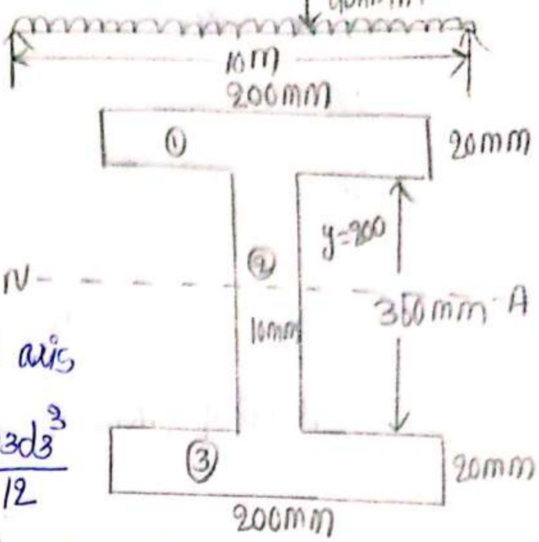
$$5\times10^6 \times L^2 = \frac{7\times10^8\times120}{250}$$

$$L^2 = \frac{7\times10^8\times120}{5\times10^6\times250} = 67.2$$

$$L = \sqrt{67.2} = 8.19\text{m}$$

6) A rolled steel Joist of I-section has the dimensions as shown in fig. This beam of I-section carries a UDL of 40kN/m run on a span of 10m. calculate the max. stress produced due to bending.

Sol) Given data



UDL "w" = 40 kN/m

$= 40\times10^3$ N/m

span of Beam "L" = 10m

moment of inertia of neutral axis

$$I = \frac{b_1d_1^3}{12} + \frac{b_2d_2^3}{12} + \frac{b_3d_3^3}{12}$$

$$= \frac{200\times20^3}{12} + \frac{10\times360^3}{12} + \frac{200\times20^3}{12}$$

$$= 39.14\times10^6 \text{ mm}^4$$

section modulus, $Z = \frac{I}{y} = \frac{39.14\times10^6}{200} = 195700 \text{ mm}^3$

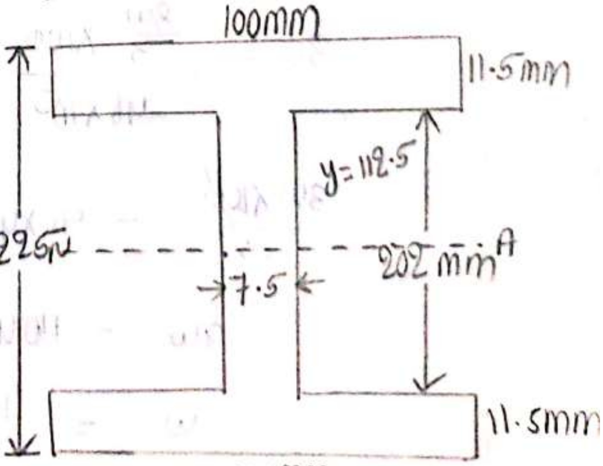
Bending moment, $m = \frac{wl^2}{8} = \frac{40\times10^3\times10^2}{8} = 5\times10^5$ Nm

$= 5\times10^8$ Nmm

$$\frac{m}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{m}{I}y = \frac{5\times10^8}{39.14\times10^6}\times200 = 2554.93 \text{ N/mm}^2$$

7) An I-section shown in fig is SSB at a point load "w" acting at a distance of 4m from right support. If the max permissible bending stress is 80 N/mm², span length is 12m.

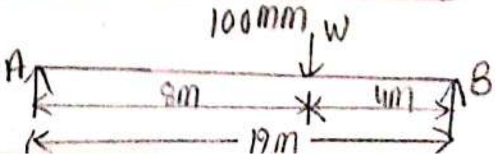


Sol) Given data

Bending stress $\sigma_b = 80$ N/mm²

Point load on SSB = W

Taking moments about A



$R_B\times12 - W\times8 = 0$

$$R_B = \frac{W8}{12} = \frac{2W}{3}$$

Sum of forces

$R_A + R_B = W$

$R_A = W - \frac{2W}{3} = \frac{W}{3}$

Bending moment at supports A & B will be zero

Bending moment at "C", $M_C = R_A \times 8$

$= \frac{W}{3} \times 8$

$= \frac{8W}{3}$ Nm

$= \frac{8W}{3} \times 1000$ Nmm

moment of inertia

$$I = \frac{b_1 d_1^3}{12} + \frac{b_2 d_2^3}{12} + \frac{b_3 d_3^3}{12}$$

$$= \frac{100 \times 11.5^3}{12} + \frac{7.5 \times 202^3}{12} + \frac{100 \times 11.5^3}{12}$$

$= 51.76 \times 10^5$ mm⁴

Section modulus, $Z = \frac{I}{y} = \frac{51.76 \times 10^5}{112.5} = 46008.88$ mm³

$= 46 \times 10^3$ mm³

$\sigma_b = \frac{M}{Z}$

$$80 = \frac{\frac{8W}{3} \times 1000}{46 \times 10^3}$$

$$\frac{8W \times 10\cancel{0}0}{3} = 80 \times 46 \times 10^{\cancel{3}}$$

$8W = 11040$

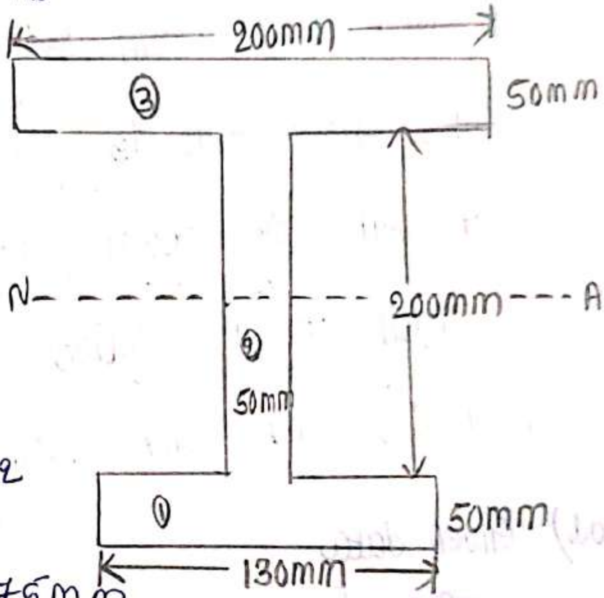
$W = \frac{11040}{8} = 1380$ N

8) A cast iron bracket subject to bending has the cross-section of I form with unequal flanges. the dimensions of the section as shown in fig. find the position of neutral axis and moment of inertia. If the B.m max is 40mNmm, Determine the max. bending stress.

sol/ Given data

max. BM, $m = 40mNmm$

$= 40 \times 10^6 \, Nmm$



$A_1 = 130 \times 50 = 6500 \, mm^2$

$y_1 = \frac{50}{2} = 25 \, mm$

$A_2 = 50 \times 200 = 1000 \, mm^2$

$y_2 = 50 + \frac{200}{2} = 150 \, mm$

$A_3 = 200 \times 50 = 10000 \, mm^2$

$y_3 = 50 + 200 + \frac{50}{2} = 275 \, mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000} = 166.50 \, mm$$

$$I_1 = \frac{130 \times 50^3}{12} + 6500 (166.5 - 25)^2 = 131.49 \times 10^6 \, mm^4$$

$$I_2 = \frac{50 \times 200^3}{12} + 10000 (166.5 - 150)^2 = 36.05 \times 10^6 \, mm^4$$

$$I_3 = \frac{200 \times 50^3}{12} + 10000 (166.5 - 275)^2 = 119.80 \times 10^6 \, mm^4$$

$I = I_1 + I_2 + I_3$

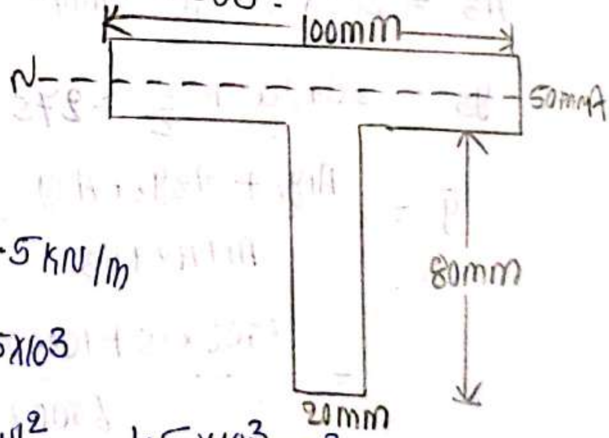
$= (131.49 + 36.05 + 119.80) \times 10^6$

$= 287.34 \times 10^6 \, mm^4$

$Z = \frac{I}{y} = \frac{287.34 \times 10^6}{166.50} = 1.72 \times 10^6 \, mm^3$

$\sigma_b = \frac{M}{Z} = \frac{40 \times 10^6}{1.72 \times 10^6} = 23.25 \, N/mm^2$

9) A cast iron beam is off T-section as shown in fig. The beam is simply supported on a span of 8m. The beam carries a UDL of 1.5 kN/m length on the entire span. Determine the max. tensile and compressive stresses.



sol) Given data

span of Beam, $L = 8m$

UDL on Beam, $w = 1.5 \, kN/m$

$= 1.5 \times 10^3$

B.M for S.S.B, $M = \frac{wL^2}{8} = \frac{1.5 \times 10^3 \times 8^2}{8}$

$= 12000 \, Nm$

$= 12000 \times 10^3 \, Nmm$

$= 12 \times 10^6 \, N \, mm$

$89.24 \, mm$

$A_1 = 20 \times 80$ $\quad y_1 = \frac{80}{2} = 40mm$

$= 1600mm^2$

$A_2 = 100 \times 50$ $\quad y_2 = 80 + \frac{50}{2} = 105mm$

$= 5000mm^2$

$$\bar{y}_{bottom} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{1600 + 40 + 5000 \times 105}{1600 + 5000}$$

$$= 89.24$$

$$I_1 = \frac{b_1d_1^3}{12} + A_1(\bar{y} - y_1)^2 = \frac{20 \times 80^3}{12} + 1600(89.24 - 40)^2$$

$$= 4.73 \times 10^6 mm^4$$

$$I_2 = \frac{b_2d_2^3}{12} + A_2(\bar{y} - y_2)^2 = \frac{100 \times 50^2}{12} + 5000(89.24 - 105)^2$$

$$= 2.28 \times 10^6 mm^4.$$

$$I = I_1 + I_2 = 4.73 \times 10^6 + 2.28 \times 10^6 = 7.01 \times 10^6 mm^4$$

$$\bar{y}_{top} = 130 - 89.24 = 40.76mm$$

$$\frac{m}{I} = \frac{f}{y}$$

compressive stress, $\sigma_c = \frac{m}{I} \times y_c$

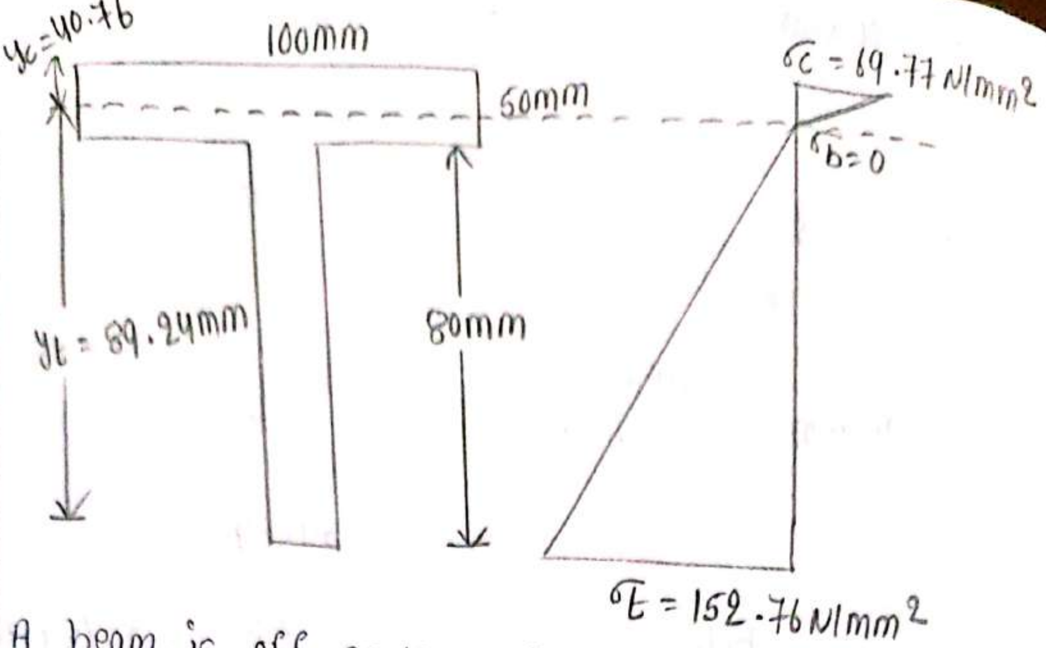
$$= \frac{12 \times 10^6}{7.01 \times 10^6} \times 40.76$$

$$= 69.77 N/mm^2$$

Tensile stress, $\sigma_t = \frac{m}{I} \times y_t$

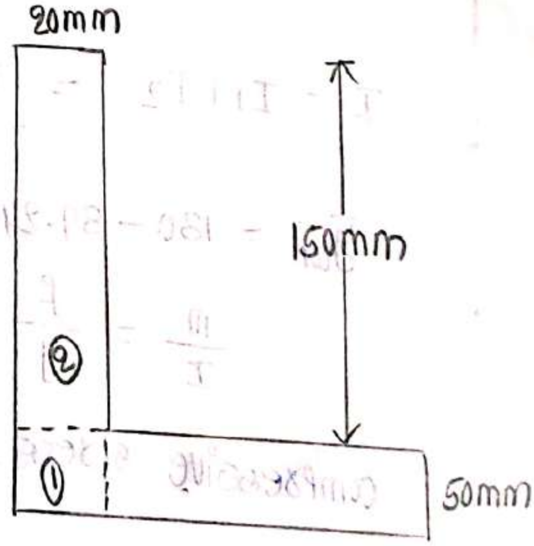
$$= \frac{12 \times 10^6}{7.01 \times 10^6} \times 89.24$$

$$= 152.76 N/mm^2$$



10) A beam is off angle section as shown in fig. The beam is SSB of length 4m carries a point load of 16kN @ a distance of 3m from the left support. Determine the max. Tensile & compressive stress of "L" section.

Sol) Given data



$A_1 = 100 \times 50 = 5000\,mm^2$

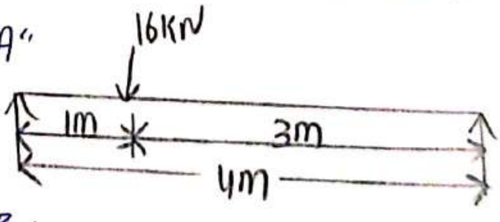
$y_1 = \frac{50}{2} = 25mm$

$A_2 = 20 \times 150 = 3000mm^2$

$y_2 = 50 + \frac{150}{2} = 125mm$

$w = 16kN = 16 \times 10^3 N$

Taking moments about "A"



$R_B \times 4 - 16 \times 1 = 0$

$R_B = 16/4 = 4kN = 4 \times 10^3 N$

Sum of forces acting on the beam

$R_A + R_B = 16 \times 10^3$

$R_A + 4 \times 10^3 = 16 \times 10^3$

$R_A = 16\times10^3 - 4\times10^3$

$R_A = 12\times10^3$

→ BM at A & B will be zero

BM at C, $M = R_A \times 1000$

$= 12\times10^3 \times 1000$

$M = 12\times10^6$ N/mm

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2} = \frac{(5000\times25)+(3000\times125)}{5000+3000}$$

$y_t = 62.5$ mm

$y = y_c + y_t$

$200 = y_c + 62.5$

$y_c = 200 - 62.5 = 137.5$ mm

$$I_1 = \frac{b_1d_1^3}{12} + A_1(\bar{y}-y_1)^2$$

$$= \frac{100\times50^3}{12} + 5000(62.5-25)^2$$

$= 8.07\times10^6$ mm^4

$$I_2 = \frac{b_2d_2^3}{12} + A_2(\bar{y}-y_2)^2$$

$$= \frac{20\times150^3}{12} + 3000(62.5-125)^2$$

$= 17.34\times10^6$ mm^4

$I = I_1 + I_2$

$= 8.07\times10^6 + 17.34\times10^6$

$= 25.41\times10^6$ mm^4

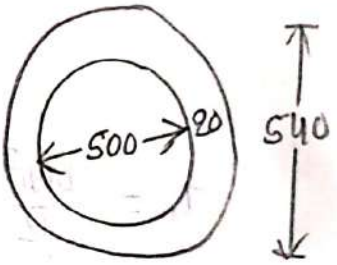
$$\sigma_c = \frac{M}{I} \times y_c \qquad \sigma_t = \frac{M}{I} \times y_t$$

$$= \frac{12 \times 10^6}{25.41 \times 10^6} \times 137.5 \qquad = \frac{12 \times 10^6}{25.41 \times 10^6} \times 62.5$$

$$= 64.93 \ N/mm^2 \qquad = 29.51 \ N/mm^2$$

11) A water main of 500mm in dia & 20mm Thickness is running full. The water main is off cast iron and is supported at 2 points 10m apart. Find the max. stress in the metal. The cast iron & water weights 72000 N/m^3 & 10000 N/m^3 respectively

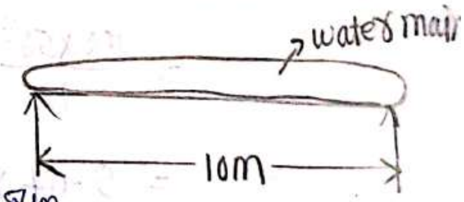
Sol) Given data



Internal Diameter "d" = 500mm = 0.5m

Thickness "t" = 20mm

External Diameter "D" = d+2t



= 500+2(20)

= 540mm = 0.54m

Density of cast iron $\rho_c = 72 \times 10^3 \ N/m^3$

Density of water $\rho_w = 10 \times 10^3 \ N/m^3$

$$Z = \frac{I}{y} = \frac{\pi}{32D}\left[D^4 - d^4\right] = \frac{\pi}{32(540)}\left[540^4 - 500^4\right]$$

$$= 4.09 \times 10^6 \ mm^3$$

$$I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[540^4 - 500^4\right]$$

$$= 1105.96 \times 10^6 \ mm^4$$

1) Inner area of pipe $= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.5^2 = 0.1960 m^2$

This is also equal to the area of water section

Area of water section $= 0.196 m^2$

2) outer area of pipe $= \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.54^2 =$

Area of pipe section $= \frac{\pi}{4} \left[D^2 - d^2 \right]$

$$= \frac{\pi}{4} (0.54^2 - 0.5^2)$$

$$= 0.0327 m^2$$

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (540^4 - 500^4) = 1.105 \times 10^9 mm^4$$

weight of pipe $= 72000 \times 0.0327 \times 1$

for 1m run $= 2354 N$

weight of water for 1m run $= 10000 \times 0.196 \times 1 = 1960 N$

Total weight $= 2354 + 1960 = 4314 N$

$$M = \frac{wL^2}{8} = \frac{4314 \times 10^2}{8} = 53925 Nm$$

$$= 53925 \times 10^3 N/m$$

$$\frac{M}{I} = \frac{\sigma}{y} \qquad y = \frac{D}{2} = \frac{540}{2}$$

$$= 270 mm$$

$$\sigma = \frac{M}{I} \times y$$

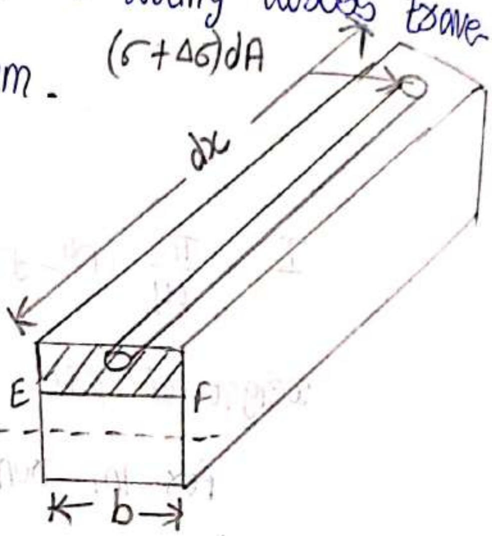
$$= \frac{53925 \times 10^3}{1.105 \times 10^9} \times 270$$

$$= 13.18 N/mm^2$$

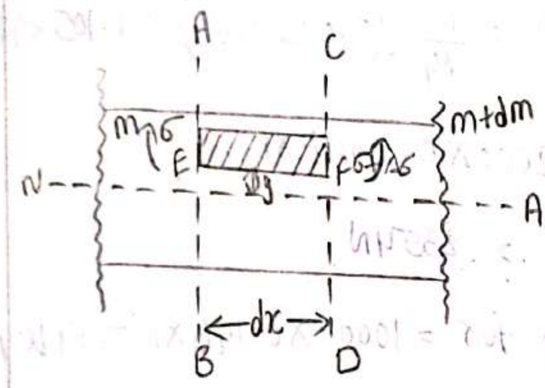
Shear stress

Def: The beam is subjected to bending moment varies section it have shearforce value, due to these shear force the beam will be subjected to shear stress.

⇒ These shear stress will be acting across transverse section of the beam.



Shear stress equation :



1) consider a simply supported beam carrying udl. for a udl the SF & BM will vary along the length of the beam.

2) consider two sections AB & CD of beam at a distance 'dx'

3) At the section AB, F = shear force

m = Bending moment

4) At the section CD, F+dF = shear force

m+dm = Bending moment

5) let us find the shear stress on the section AB at a distance y from the NA.

on the cross-section of the beam - let EFBA line @ a distance "y" from the NA. Now consider the part of the beam above the level EF & between the sections AB & CD.

6) This part of beam consists of an infinite no. of layers or elements each of area dA & length dx.

7) consider one element at a distance of y from NA.

8) The bending stress @ distance "y" from NA at section AB.

$$\frac{m}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{m}{I} \cdot y \rightarrow ①$$

The bending stress at distance "y" from NA at the section ①

$$\frac{m+dm}{I} \times y = (\sigma + \Delta\sigma) \rightarrow ②$$

9) The shear force at the section AB on the elemental area

$$F = (\sigma \times dA$$

$$F = \frac{m}{I} \times y \times dA$$

The shear force at the section ①

$$(F + \partial F) = (\sigma + \Delta\sigma) \times dA$$

$$= \frac{(m+dm)}{I} \cdot y \times dA \rightarrow ③$$

eq ② & ③ ⇒ These two forces acting in opposite directions.

10) net unbalanced shear force on the element

$$(\cancel{f}+df)-\cancel{f} = \frac{m+dm}{I} \times y \times dA - \frac{m}{I} \times y \times dA$$

$$df = \cancel{\frac{m}{I} y dA} + \frac{dm}{I} y dA - \cancel{\frac{m}{I} y dA}$$

$$dF = \frac{dm}{I} y dA$$

Total shear force on the beam, By using integration.

$$\int \cancel{d}f = \int \frac{dm}{I} y dA$$

$$F = \frac{dm}{I} \int y dA$$

$$F = \frac{dm}{I} \cdot \bar{y} \cdot A \rightarrow ④$$

shear force on the beam due to shear force

F = shear stress x shear Area

$$F = \tau \times (b \times dx) \rightarrow ⑤$$

equating ④ & ⑤

$$\tau (b \times dx) = \frac{dm}{I} \times \bar{y} \times A$$

$$\tau = \frac{dm}{I} \times \bar{y} \times \frac{A}{b \times dx}$$

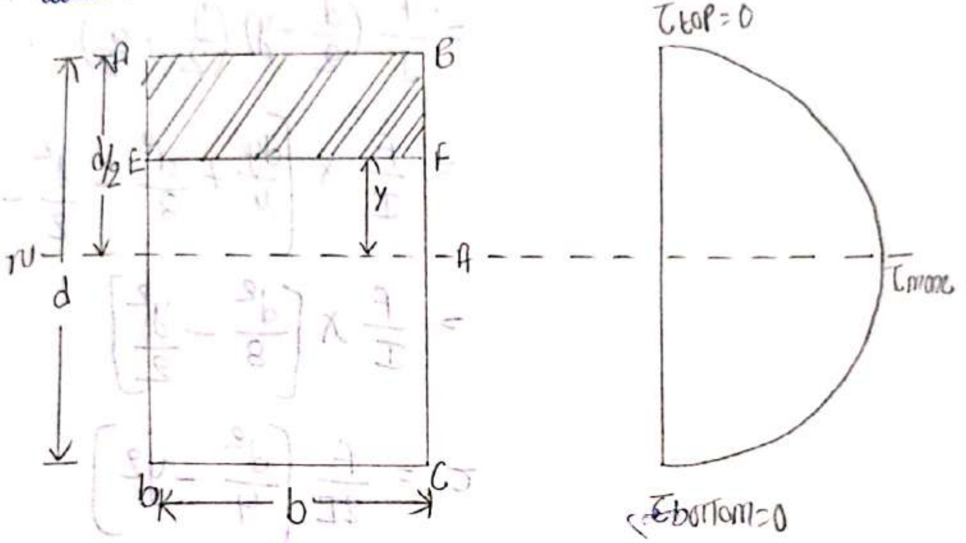
$$\tau = \frac{dm}{dx} \times \frac{A\bar{y}}{I \times b}$$

$$\tau = F \times \frac{A\bar{y}}{I \times b}$$

$\bar{y}$ = Distance from centroid of the element to the neutral axis of the section.

Shear stress distribution for different sections:

(i) Rectangular section



Consider a rectangular section of a beam of width (b) and depth (d) let "F" is the shear force acting at the section. Consider a level "EF" at a distance of "y" from the neutral axis.

The shear stress at the level "EF" $\tau = \frac{F \times A\bar{y}}{I \times b}$ → ①

A = c/s area of shaded portion

$= b \times \left(\frac{d}{2} - y\right)$

$\bar{y}$ = Distance of C·G of the area A from NA.

$$= y + \frac{\left(\frac{d}{2} - y\right)}{2}$$

$$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) \Rightarrow y + \frac{d}{4} - \frac{y}{2}$$

$$= \left(\frac{y}{2} + \frac{d}{4}\right)$$

Substitute A, $\bar{y}$ values in equation ①

$$\text{shear stress } \tau = \frac{F}{IAb} \times \left[b \times \left(\frac{d}{2} - y\right)\left(\frac{y}{2} + \frac{d}{4}\right)\right]$$

$$= \frac{F}{I}\left(\frac{d}{2} - y\right)\left(\frac{y}{2} + \frac{d}{4}\right)$$

$$= \frac{F}{I} \times \left[\cancel{\frac{dy}{4}} + \frac{d^2}{8} - \frac{y^2}{2} - \cancel{\frac{yd}{4}}\right]$$

$$= \frac{F}{I} \times \left[\frac{d^2}{8} - \frac{y^2}{2}\right]$$

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]$$

At the top edge "$y = d/2$"

$$\text{shear stress } \tau_{TOP} = \frac{F}{2I} \times \left(\frac{d^2}{4} - \frac{d^2}{4}\right)$$

$$\tau_{TOP} = 0$$

At the N·A $y = 0$

$$\tau_{NA} = \frac{F}{2I} \times \left(\frac{d^2}{4} - 0\right)$$

$$= \frac{F}{2I} \times \frac{d^2}{4}$$

$$\tau_{NA} = \frac{F}{I} \times \frac{d}{8}$$

$$= \frac{F}{bd^3/12} \times \frac{d^2}{8} \Rightarrow \frac{Fd^2 \times 12}{8\, bd^3} \Rightarrow \tau = \frac{3}{2} \times \frac{F}{bd}$$

At N.A; the shear stress is maximum

$$\tau_{max} = \frac{3}{2} \times \frac{F}{bd}$$

$$\tau_{max} = 1.5 \times \tau_{avg}$$

Average shear stress $\tau_{avg} = \frac{F}{bd}$

1) A Rectangular Beam 100mm wide & 250mm deep is subjected to a maximum shear force of 50kN.
(i) Determine avg shear stress (ii) max shear stress
(iii) shear stress at a distance of 25mm above the neutral axis.

Sol) Given data

Breadth of the Beam (b) = 100mm

Depth of the Beam (d) = 250mm

shear force (F) = 50kN
$= 50 \times 10^3 N$

i) Average shear stress $\tau_{avg} = \frac{F}{bd} = \frac{50 \times 10^3}{100 \times 250}$

$= 2 N/mm^2$

ii) max. shear stress $\tau_{max} = \frac{3}{2} \times \tau_{ave} = \frac{3}{2} \times 2$

$= 3 N/mm^2$

iii) At a distance of 25mm = y

shear stress "τ" $= \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$

$$= \frac{50 \times 10^3 \times 12}{2 \times 100 \times 250^3} \times \left(\frac{250^3}{4} - 25^2\right)$$

$$= 2.89 \, N/mm^2$$

2) 6.5 wooden beam of span 1.3m having a cross section 150mm & 250mm deep carries a point load "W" at the centre. The permissible stress are $7 N/mm^2$ in bending & $1 N/mm^2$ in shearing. calculate the safe load "W".

Sol) Given data

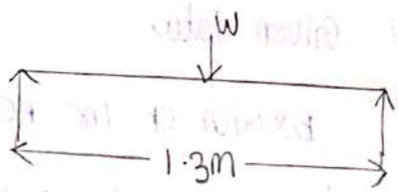
simply supported of wooden beam length $= 1.3m = 1300mm$

Breadth of The Beam ("b") $= 150 mm$

Depth of The Beam ("d") $= 250mm$

Load $=$ "W"

Bending stress (σ) $= 7 N/mm^2$



shear stress "τ" $= 1 N/mm^2$

safe load $= W \Rightarrow ?$

Bending moment for simply supported beam with point load

$$m = \frac{WL}{4} \Rightarrow m = \frac{W \times 1300}{4} = 325W, N\text{-}m$$

(i) By using bending equation

$$\frac{m}{I} = \frac{\sigma f}{y} \Rightarrow \frac{325W}{bd^3/12} = \frac{7}{d/2}$$

$$= \frac{325W}{\frac{150 \times 250^3}{12}} = \frac{7}{125}$$

$$\frac{325 \times w \times h/2}{160 \times 250^3} = \frac{f}{125}$$

$w = 33652.8 \, N$

(ii) shear force for simply supported beam with point load

$P = w/2$ shear stress $\tau_{max} = \frac{3}{2} \times \frac{F}{bd}$

$$1 = \frac{3}{2} \times \frac{\frac{w}{2}}{150 \times 250}$$

$$1 \times \frac{3}{4} \times \frac{w}{150 \times 250}$$

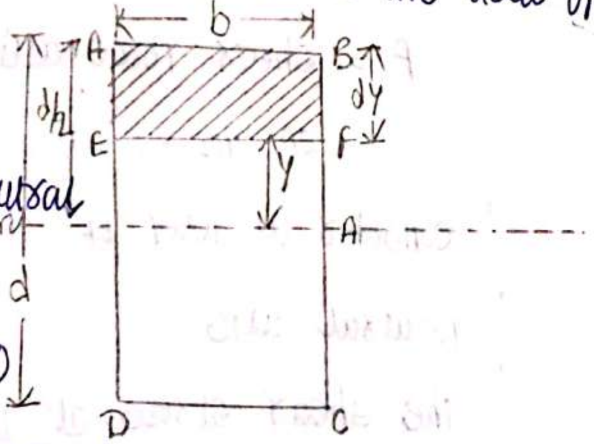
$$w = 50 \times 10^3 \, N$$

safe load "w" $= 33652.8 \, N$

<u>moment area method</u>

consider a small strip of thickness "dy" at a distance of "y" from the neutral axis let dA is the area of the strip.

$dA = b \times dy$



moment of area (dA) from neutral axis. $dA \cdot y = (dA) \times y$

$$dA \cdot y = (b \times dy) \times y \rightarrow (1)$$

the moment of a shaded area about N.A is obtained by integrating the above equation b/w the limits "y" to "d/2"

$$\int dA \cdot y = \int_y^{d/2} (b \times dy)\, y$$

$$A\bar{y} = \int_y^{d/2} (b \times dy)\, y.$$

$$= b \int_{y}^{d/2} y \times dy$$

$$= b\left(\frac{y^2}{2}\right)_{y}^{d/2} \Rightarrow b\left(\frac{(d/2)^2}{2} - \frac{y^2}{2}\right)$$

$$A\bar{y} = b\left(\frac{d^2}{8} - \frac{y^2}{2}\right)$$

shear stress $\tau = \frac{FA\bar{y}}{I \times b}$

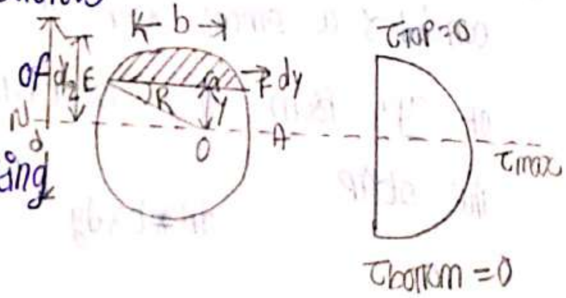
$$= \frac{F}{Ib} \times b\left(\frac{d^2}{8} - \frac{y^2}{2}\right)$$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

circular section

consider a circular section of a beam, let R is the radius of the circular section of d

F = shear force acting on the section.



consider a level "EF" at a distance "y" from the neutral axis.

The shear stress at this level $\tau = \frac{F \times A\bar{y}}{Ib} \rightarrow (1)$

consider a strip of thickness "dy" at a distance "y" from the neutral axis let "dA" is the area of strip/element.

The area at the level "EF" "dA" $= b \times dy$

$$= EF \times dy$$

$$= 2 \times EG \times dy$$

Applying pythagoras theorem : ΔOEG, $OE^2 = OG^2 + EG^2$.

$$R^2 = y^2 + EG^2$$

$$EG = \sqrt{R^2 - y^2}$$

substitute "EG" in above equation

$$dA = 2 \times \sqrt{R^2 - y^2} \times dy$$

moment of strip area from NA

$$= dA \times y$$

$$= 2 \times \sqrt{R^2 - y^2} \times dy \times y$$

$$= (2y)\sqrt{R^2 - y^2} \times dy$$

Total moment area of the circular section

$$A\bar{y} = \int_y^R (2y)\left(\sqrt{R^2 - y^2}\right) dy$$

$$A\bar{y} = -\int_y^R -(2y)\left(R^2 - y^2\right)^{1/2} dy$$

$$A\bar{y} = \left[\frac{\left(R^2 - y^2\right)^{1/2+1}}{1/2+1}\right]_y^R$$

$$= -\left[\frac{\left(R^2 - y^2\right)^{3/2}}{3/2}\right]_y^R$$

$$= -\frac{2}{3}\left[\left[R^2 - R^2\right]^{3/2} + \left(R^2 - y^2\right)\right]^{3/2}$$

$$\boxed{A\bar{y} = \frac{2}{3} \times \left(R^2 - y^2\right)^{3/2}}$$

$$= \frac{\left(R^2 - y^2\right)^{1/2+1}}{\left(R^2 - y^2\right)^{1/2}}$$

$$= \frac{\left(R^2 - y^2\right)^{\cancel{1/2}} - \left(R^2 - y^2\right)}{\cancel{\left(R^2 - y^2\right)^{1/2}}} = \left(R^2 - y^2\right)$$

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substitute $A\bar{y}$ in equation ①:

$$\tau = \frac{F}{I \times b} \times \frac{2}{3}(R^2 - y^2)^{3/2} = \frac{2F(R^2-y^2)^{3/2}}{3I \times 2 \times \sqrt{R^2-y^2}}$$

$$= \frac{F}{3I}(R^2 - y^2)$$

$$\tau = \frac{F}{3I}(R^2 - y^2)$$

At the top edge $y = d/2$

$$\text{shear stress "}\tau\text{"} = \frac{F}{3I} \times \left(\left(\frac{d}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right)$$

$$= 0$$

At NA : $y = 0$

$$\tau_{max} = \frac{F}{3I}(R^2 - 0)$$

$$= \frac{F \times (d/2)^2}{3 \times \frac{\pi}{64} d^4}$$

$$\tau_{max} = \frac{F \times \cancel{d^2} \times \cancel{64}^{16}}{\cancel{4} \times 3 \times \pi \times \cancel{d^4}}$$

$$\tau_{max} = \frac{16}{3} \times \frac{F}{\pi d^2}$$

$$= \frac{\cancel{16}}{3} \times \frac{F}{\pi \times \cancel{4} R^2}$$

$$\tau_{max} = \frac{4}{3} \frac{F}{\pi R^2}$$

$$\tau_{max} = \frac{4}{3} \tau_{ave} \Rightarrow \tau_{avg} = \frac{F}{\pi R^2}$$

1) A circular beam of 100mm diatan diameter is subjected to a shear force of 5kN calculate.

i) Avg shear stress (ii) max, shear stress (iii) shear stress at a distance of 40mm from the neutral axis.

Sol) Given data

diameter of a circle "D" = 100mm

shear force = 5kN = 5×10^3 N/m

i) Avg shear stress:

$$\tau_{avg} = \frac{F}{\pi R^2} = \frac{5\times10^3}{\pi \times \left(\frac{d}{2}\right)^2} = \frac{5\times10^3}{\pi \times (50)^2}$$

$$= 0.636\ N/mm^2$$

ii) max. shear stress:

$$\tau_{max} = \frac{4}{3} \times \tau_{avg}$$

$$= \frac{4}{3} \times 0.636$$

$$= 0.848\ N/mm^2$$

iii) shear stress at a distance of 40mm from the NA.

$$\tau = \frac{F}{3I}(R^2 - y^2)$$

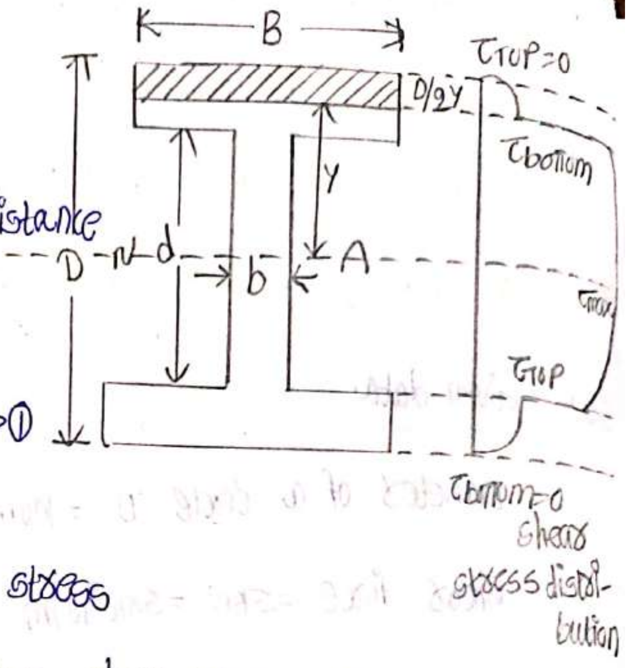
$$= \frac{50\times10^3}{3 \times \frac{\pi d^4}{64}}\left((50)^2 - (40)^2\right)$$

$$= 3.0557\ (or)\ 0.305\ N/mm^2$$

I- section

consider I-section

the shear stress at a distance of "y" from NA.



$$\tau = F \frac{A\bar{y}}{I \times b} \rightarrow (1)$$

In this case, the shear stress distribution in the web & shear stress distribution in the flange are to be calculated separately

i) shear stress distribution in the flange

consider a section at a distance of "y" from the neutral axis.

width of the beam $= B$

Area of the strip $A = \left(\frac{D}{2} - y\right) \times B$

$$\bar{y} = y + \frac{\left(\frac{D}{2} - y\right)}{2}$$

$$\bar{y} = y + \frac{D}{2} - \frac{y}{2}$$

$$\bar{y} = \frac{y}{2} + \frac{D}{4}$$

substitute $A, \bar{y}$ values in eq (1):

$$\tau = \frac{F}{I \times b} \times \left(\frac{D}{2} - y\right) \times B\left(\frac{D}{4} \times y/2\right)$$

$$\tau = \frac{F}{I} \times \left(\frac{D}{2} - y\right)\left(\frac{D}{4} + y/2\right)$$

$$\tau = \frac{F}{I}\left(\frac{D}{2} - y\right) \times \frac{1}{2}\left(\frac{D}{2} + y\right)$$

$$= \frac{F}{2I} \times \left[\left(\frac{D}{2}\right)^2 - y^2\right]$$

$$\tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$

At the top of the flange "y" $= D/2$

$$\tau_{Top} = \frac{F}{2I} \times \left(\frac{D^2}{4} - \frac{D^2}{4}\right) = 0$$

At the bottom of the flange $y = d/2$

$$\tau_{bottom} = \frac{F}{2I}\left(\frac{D^2}{4} - \frac{d^2}{4}\right)$$

$$\tau_{bottom} = \frac{F}{8I}\left(D^2 - d^2\right)$$

At the NA ; $y = 0$

maximum shear stress

$$\tau_{max} = \frac{F}{2I} \times \left(\frac{D^2}{4} - 0\right)$$

$$\tau_{max} = \frac{FD^2}{8I}$$

i) <u>shear stress distribution in the web</u> :

consider a section at a distance "y" in the web from the neutral axis.

width of section $= b$

shear stress $\tau = F \times \frac{A\bar{y}}{I \times b} \rightarrow ①$

$A\bar{y}$ is made up of two parts that is movement of flange area at movement of the shaded area of the web from the NA.

$$= B \times \left(\frac{D}{2} - \frac{d}{2}\right)\left[\frac{\left(\frac{D}{2} - \frac{D}{2}\right)}{2} + d/2\right] + \left[\left(b \times \frac{d}{2} - y\right)\left(y + \frac{\left(\frac{d}{2} - y\right)}{2}\right)\right]$$

$$A\bar{y} = \left[\left(B \times \left(\frac{D}{2} - d/2\right)\left(\frac{D}{4} - \frac{d}{4} + d/2\right)\right] + \left[b \times \left(\frac{d}{2} - y\right)\left(y + \frac{d}{4} - \frac{y}{2}\right)\right]$$

$$= \left[\left(B \times \left(\frac{D}{2} - d/2\right)\right)\left(\frac{D}{4} + d/4\right)\right] + \left[b \times \left(\frac{d}{2} - y\right)\left(\frac{d}{2} + \frac{y}{2}\right)\right]$$

$$= \left[\frac{B}{2} \times \left(\frac{D}{2} - d/2\right)\left(\frac{D}{2} + d/2\right)\right] + \left[\frac{b}{2}\left(\frac{d}{2} - y\right)\left(\frac{d}{2} + y\right)\right]$$

$$= \left[\frac{B}{2} \times \left(\frac{D^2}{4} - \frac{d^2}{4}\right)\right] + \left[\frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

$$A\bar{y} = \frac{B}{8}\left(D^2 - d^2\right) + \frac{b}{2}\left[\frac{d^4}{4} - y^2\right]$$

shear stress in the web "τ" $= \frac{F \times A\bar{y}}{I \times b}$

$$= \frac{F}{I \times b} \times \frac{B}{8}\left(D^2 - d^2\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

At the NA $y = 0$;

$$\tau_{max} = \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

$$\tau = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2)\right] + \frac{bd^2}{8}$$

At the Junction of the flange & web

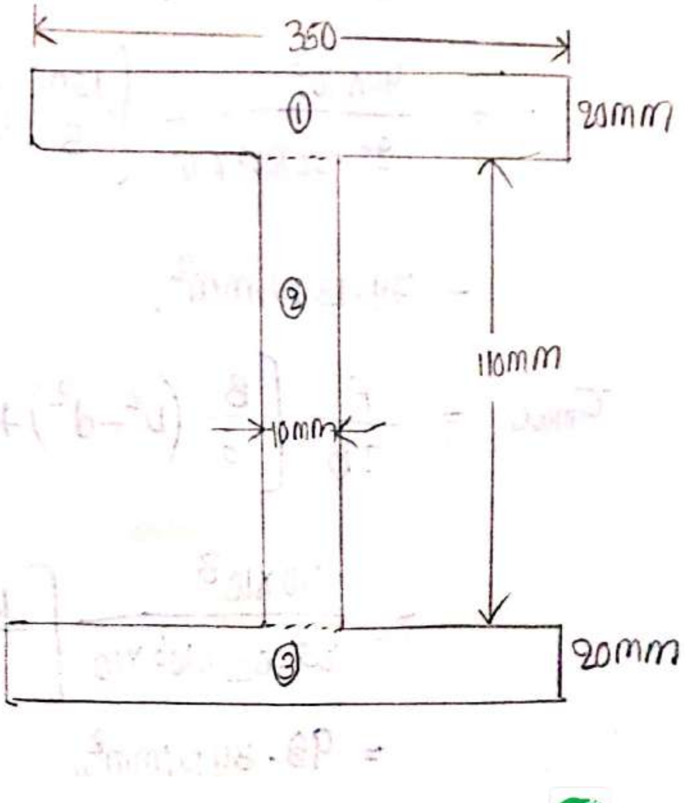
$$y = d/2$$

$$\tau_{Junction} = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + b/2\left(\frac{d^2}{4} - \frac{d^2}{4}\right)\right]$$

$$= \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2)\right]$$

1) An I-section beam 350 x 150mm web thickness of 10mm & a flange thickness of 20mm. If the shear force acting on the section is 40kN. find the max. shear stress developed in the I-section & sketch the shear stress distribution.

Sol) Given data



shear force "F" = 40kN = 40×10^3 N

moment of inertia,

$$I = \frac{b_1 d_1^3}{12} + \frac{b_2 d_2^3}{12} + \frac{b_3 d_3^3}{12}$$

$$= \frac{150\times20^3}{12} + \frac{10\times310^3}{12} + \frac{150\times20^3}{12}$$

$$= 25.02\times10^6 \text{ mm}^4$$

<u>shear stress Distribution in flange</u>:

$$\tau_{max} = \frac{FD^2}{8I} = \frac{40\times10^3\times350^2}{8\times25.02\times10^6} = 24.48 \text{ N/mm}^2$$

$$\tau_{bottom} = \frac{F}{8I}\left(D^2-d^2\right) = \frac{40\times10^3}{8\times25.02\times10^6}\left(350^2-310^2\right)$$

$$= 5.27 \text{ N/mm}^2$$

<u>shear stress Distribution in the web</u>:

$$\tau_{Jun} = \frac{F}{Ib}\left[\frac{B}{8}\left(D^2-d^2\right)\right] =$$

$$= \frac{40\times10^3}{25.02\times10^6\times10}\left[\frac{150}{8}\left(350^2-310^2\right)\right]$$

$$= 79.13 \text{ N/mm}^2$$

$$\tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}\left(D^2-d^2\right)+\frac{bd^2}{8}\right]$$

$$= \frac{40\times10^3}{25.02\times10^6\times10}\left[\frac{150}{8}\left(350^2-310^2\right)+\frac{10\times3}{8}\right.$$

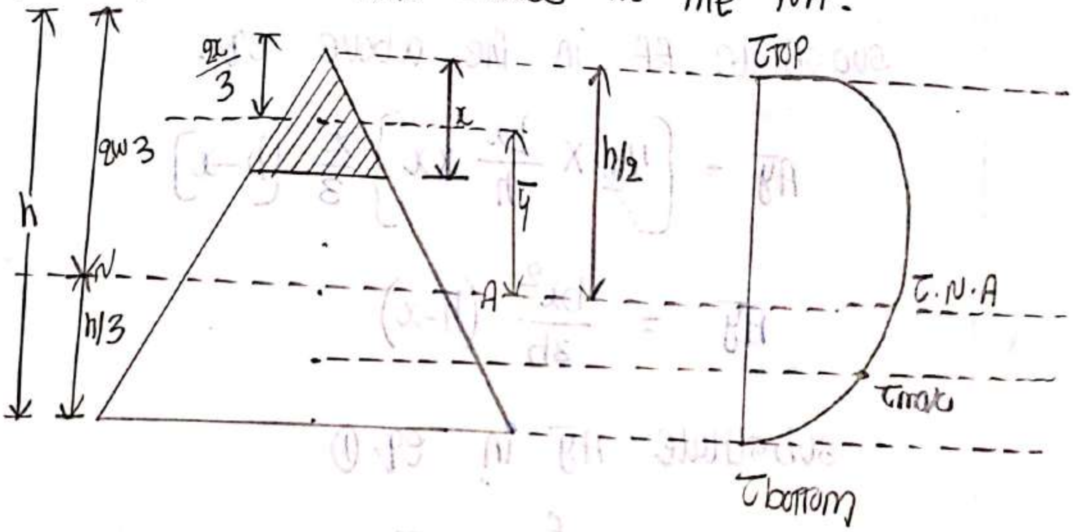
$$= 98.34 \text{ N/mm}^2$$

$$\tau_{sum} = \frac{F}{Ib}\left[\frac{B}{8}(D^2-d^2)\right]$$

$$= \frac{40 \times 10^3}{1.57}$$

Triangular section

The shear force acting on a beam @ a section F. The section of the beam is Δ are base-b & height "h". The beam is placed with it's base horizontal. Find the max. shear stress at the NA.



1) The NA of the Δ ABC will lie at the C.G of the Δ but the C.G of the Δ will be located at a distance of $2h/3$ from the top or apex.

2) consider a level EF at a distance of y from the NA, the shear stress at this level $\tau = F\frac{A\bar{y}}{Ib} \rightarrow ①$

3) consider a small strip Δ CEF at a distance of "x" from apex.

$A\bar{y}$ = shaded area of ΔCEF x Distance from NA to CG of the ΔCEF.

$$= \left[\frac{1}{2} \times EF \times x\right]\left[\frac{2h}{3} - \frac{2x}{3}\right]$$

$$= \left[\frac{1}{2} \times EF \times x\right] \frac{2}{3}(h-x)$$

similar triangles ΔCEF, ΔABC

$$\frac{EF}{AB} = \frac{x}{h}$$

$$EF = \frac{x}{h} \times b$$

substitute EF in the above eq.

$$A\bar{y} = \left[\cancel{1/2} \times \frac{bx}{h} \times x\right] \frac{\cancel{2}}{3}\left[h-x\right]$$

$$A\bar{y} = \frac{bx^2}{3h}(h-x)$$

substitute $A\bar{y}$ in eq ①

$$\tau = \frac{F}{Ib} \times A\bar{y}$$

$$= \frac{F}{Ib} \times \frac{bx^2}{3h}(h-x)$$

$$= \frac{F}{I \times EF} \times \frac{bx^2}{3h}(h-x)$$

$$= \frac{F}{I \times \frac{\cancel{x}}{\cancel{h}} \times \cancel{b}} \times \frac{\cancel{b}x^{\cancel{2}}}{3\cancel{h}}(h-x)$$

$$\boxed{\tau = \frac{Fx}{3I}(h-x)} \rightarrow ②$$

At the apex, $x = 0$; $\tau_{top} = 0$

At the NA, $x = \frac{2h}{3}$; $\tau_{NA} = \frac{F \times 2h/3}{3I}\left[h - \frac{2h}{3}\right]$

$$= \frac{2Fh}{9I} \times \frac{h}{3}$$

$$= \frac{2F\cancel{h^2}}{27 \times \frac{b\cancel{h^3}}{36}} = \frac{2 \times 36}{27} \times \frac{F}{bh}$$

$$\tau_{NA} = \frac{8F}{3bh}$$

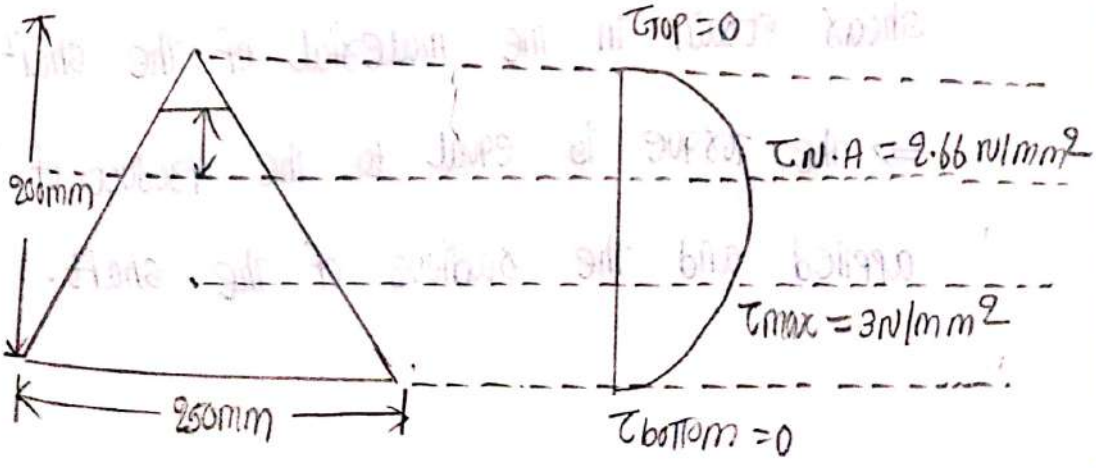
At the centre, $x = h/2$; $\tau_{max} = \frac{F \times h/2}{3 \times \frac{bh^3}{36}}\left[h - h/2\right]$

$$= \frac{F \times \cancel{36}^{\cancel{12}\,3} \times \cancel{h^2}}{\cancel{3} \times b\cancel{h^3} \times \cancel{4}}$$

$$= \frac{3F}{bh}$$

1) A beam of triangular cross-section is subjected to a shear force of 50kN. The base width of section is 250mm & height is 200mm. The beam is placed with it's base horizontal. find τ_{max} & τ_{NA}.

Sol) Given data



shear force, $F = 50\,kN = 50 \times 10^3\,N$

Base width "b" = 250mm

height, "h" = 200mm

i) $\tau_{max} = \frac{3F}{bh} = \frac{3 \times 50 \times 10^3}{250 \times 200} = 3\,N/mm^2$

ii) $\tau_{NA} = \frac{8F}{3bh} = \frac{8 \times 50 \times 10^3}{3 \times 250 \times 200} = 2.67\,N/mm^2$

Question for circular section derivation

prove that the max. shear stress in a circular section of a beam is 4/3 times the average shear stress.

TORSION

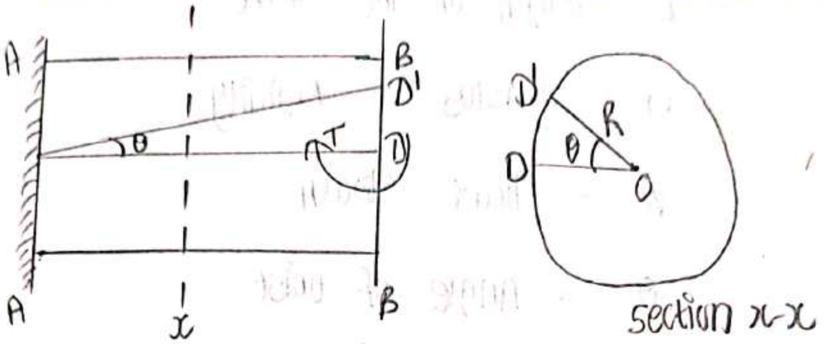
Twisting moment

A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shafts. due to the application of the torques at the two ends, the shaft is subjected to a twisting moment this causes the shear stresses and shear strain in the material of the shaft.

⇒ the torque is equal to the product of force applied and the radious of the shaft.

Torsion Equation :

This equation is also known as shear stress equation.



1) Derivation of shear stress produced in a circular shaft subjected to a Torsion.

2) consider a circular shaft which is subjected to a torsion, shear stresses are setup in the material of the shaft. To determine the equation of shear stress at any point on the shaft.

3) consider a circular shaft fixed at one end A-A and free at other end B-B.

4) let CD is any line on the outer surface of the shaft. now let the shaft is subjected to a torque 'T' at the end B-B, will rotate clockwise and every cross-section of the shaft will be subjected to shear stresses.

5) The point D will shift to D' and the line CD will be deflected to CD' with an angle ϕ.

let. R = Radious of the shaft

L = length of the shaft

G = modulus of Rigidity

ϕ = shear strain

θ = Angle of twist

shear. strain at outer surface

$$\phi = \frac{DD'}{L} \rightarrow (1)$$

section x-x; Angle of twist $\theta = \frac{DD'}{OD}$

$$\theta = \frac{DD'}{R}$$

$$DD' = R\theta$$

substitute DD' in eq (1)

$$\phi = \frac{R\theta}{L}$$

modulus of Rigidity "G" = $\frac{\text{shear stress}}{\text{shear strain}}$

$$G = \frac{\tau}{\phi}$$

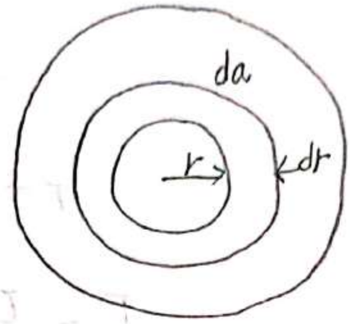
$$G = \frac{\tau}{\frac{R\theta}{L}}$$

$$\frac{G\theta}{L} = \frac{\tau}{R} \rightarrow (2)$$

Area of small strip $da = 2\pi r \cdot dr$

shear stress on the element/strip

$$\frac{\tau}{R} = \frac{q}{r}$$



$$q = \frac{\tau}{R} r$$

$$q = \tau \cdot \frac{r}{R}$$

shear force on element, dF = shear stress on the element × Elemental area

$$= q \times da$$

$$= \tau \cdot \frac{r}{R} \times 2\pi r \cdot dr$$

$$= \frac{2\pi\tau}{R} r^2 dr$$

Twisting moment on the element,

dT = shear force on the element × Distance

$$= dF \times r$$

$$= \frac{2\pi\tau}{R} r^2 dr \times r$$

$$= \frac{2\pi\tau}{R} r^3 dr$$

By using Integration,

$$\int dT = \int_0^R \frac{2\pi\tau}{R} r^3 \, dr$$

$$T = \frac{2\pi\tau}{R} \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2\pi\tau}{R} \times \frac{R^4}{4}$$

$$= \frac{\cancel{2}\pi\tau}{R} \times \frac{(D/2)^4}{\cancel{4}\,2}$$

$$= \frac{\tau}{R} \times \frac{\pi D^4}{32}$$

$$T = \frac{\tau}{R} \times J$$

$$\frac{T}{J} = \frac{\tau}{R} \rightarrow ③$$

Equating ② & ③, we get

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$ → this is called as the torsional equation.

<u>Polar section modulus</u>

It is defined as the ratio of the polar moment of inertia to the radius of the shaft. It is also known as polar modulus or torsional section modulus. It is denoted by Z_P.

→ <u>for solid shaft</u> : $Z_P = \frac{J_P}{R}$

$$= \frac{\frac{\pi D^4}{\cancel{32}\,16}}{D/\cancel{2}}$$

$$\boxed{Z_P = \frac{\pi D^3}{16}}$$

→ for hollow shaft : $Z_P = \frac{J_P}{R}$

$$= \frac{\frac{\pi}{\cancel{32}\,16}\left(D^4 - d^4\right)}{D/\cancel{2}}$$

$$\boxed{Z_P = \frac{\pi(D^4-d^4)}{16D}}$$

Torsional Rigidity

It is defined as the torque required to produce a twist of 1 radian per unit length of the shaft. It is denoted by T.

from torsion equation, $\frac{T}{J} = \frac{G\theta}{L}$ where $\theta = 1°$

$L = 1m$

$$\frac{T}{J} = \frac{G \times 1°}{1}$$

Radians $R = \theta \times \frac{\pi}{180°}$

$$\boxed{T = GJ}$$

Degrees $\theta = R \times \frac{180°}{\pi}$

Power transmitted by shafts: units: watts.

$P = \frac{2\pi NT}{60}$; N = number of revolutions in rpm (rotates 1 minute)

T = Torque in N-m

m = moment in N-mm

P = power in watts.

(1) Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 rpm. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The max. shear stress is limited to 60 N/mm². Take $G = 8 \times 10^4$ N/mm².

Sol) Given data

Power transmitted "P" = 90 kW = 90×10^3 W

no. of revolutions "N" = 160 rpm

Angle of twist "θ" = $1°$ = 0.017 radians

max. shear stress "τ" = 60 N/mm²

modulus of rigidity 'G' = 8×10^4 N/mm²

(i) Diameter of the shaft (d):

From Torsion Equation,

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{T}{\pi d^4/32} = \frac{\tau}{d/2}$$

$$T = \frac{2\pi N T}{60}$$

$$T = \frac{90 \times 10^3 \times 60}{2\pi \times 160} = 5371.47 \text{ N-m} = 5371.47 \times 10^3 \text{ N-mm}$$

$$\frac{5371.47 \times 10^3 \times 32}{\pi d^4} = \frac{60 \times 2}{d}$$

$$\frac{5371.47 \times 10^3 \times 32}{\pi \times 60 \times 2} = d^3$$

$$d = \sqrt[3]{455944.5} = 76.96 \text{mm}$$

(ii) Length of the shaft "L":

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\frac{60}{38.48} = \frac{8 \times 10^4 \times 0.017}{L}$$

$$= 872.21 \text{mm}$$

2) A hollow shaft of dia ratio 3/8 is to transmit 375 KW power at 100 rpm. The max. torque being 20% greater than the mean. The shear stress is not to exceed 60 N/mm² and twist in a length of 4m not to exceed 2°. calculate it's external and internal diameters which wood satisfy both the above conditions. Assume "G" = 0.85 X 105 N/mm².

sol) Given data

Diameter Ratio, $\frac{d}{D} = \frac{3}{8} \Rightarrow d = \frac{3}{8} D$

power transmitted, "P" = 375 KW = 375×10^3 W

no. of revolutions, "N" = 100 rpm

max. shear stress, τ = 60 N/mm²

Angle of twist, $\theta = 2° \times \frac{\pi}{180} = 0.034$ radians

length of shaft, "L" = 4m = 4000mm

modulus of rigidity, "G" = 0.85×105 N/mm²

max. torque = 20% of Tmean = 100% + 20% of Tmean

T_{max} = 120% of T_{mean}

$$P = \frac{2\pi N T_{mean}}{60}$$

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{375 \times 10^3 \times 60}{2\pi \times 100} = 35809.86 \text{ N-m}$$

$$T_{max} = \frac{120}{100} \times 35809.86 = 42971.83 \text{ N-m}$$

$$= 42971.83 \times 10^3 \text{ N-mm}$$

case-1

Diameter of The shaft when The shear stress is not to exceed 60 N/mm². $\Rightarrow \frac{T}{J} = \frac{\tau}{R}$

case-2

$$\frac{T}{J} = \frac{G\theta}{L}$$

(i) $\frac{T}{J} = \frac{\tau}{R}$

$$\frac{42917.83 \times 10^3}{\frac{\pi}{32}\left[D^4 - \left(\frac{3}{8}d\right)^4\right]} = \frac{60}{D/2}$$

$$\frac{42971.83 \times 10^3 \times 32}{\pi D^4\left[1 - \frac{81}{4096}\right]} = \frac{60 \times 2}{D}$$

$$D^3 = \frac{42971.83 \times 10^3 \times 32}{120 \times \pi \times \left[1 - \frac{81}{4096}\right]}$$

$$D = \sqrt[3]{3721149.701} = 154.96\,mm$$

outer Diameter of The shaft $D = 154.96\,mm$

Inner Diameter of The shaft $d = \frac{3}{8}(154.96)$

$= 58.11\,mm$

(ii) $\frac{T}{J} = \frac{G\theta}{L}$

$$\frac{42971.83 \times 10^3}{\frac{\pi}{32}(D^4 - d^4)} = \frac{0.85 \times 10^5 \times 2^\circ \times \frac{\pi}{180^\circ}}{4 \times 10^3}$$

$$= \frac{42971.83 \times 10^3 \times 32}{\pi D^4\left(1 - \frac{81}{4096}\right)} = 0.7417$$

$$D^4 = \frac{42971.83\times10^3\times32}{\pi D^4\left[1-\frac{81}{4096}\right]} = 0.7417$$

$$D^4 = \frac{42971.83\times10^3\times32}{0.7417\times\pi\left(1-\frac{81}{4096}\right)} = 602046552.6$$

$$D = \sqrt[4]{602046552.6} = 156.64\,mm$$

$$d = \frac{3}{8}(156.64) = 58.74\,mm$$

(3) A solid circular shaft transmits 75kW power at 200rpm. calculate the shaft Dia if the twist in the shaft is not to exceed 1° in 2m length of shaft & shear stress $15\,N/mm^2$. Take $G = 1\times10^5\,N/mm^2$.

Sol) Given data

Power transmitted, "P" = 75 kW = $75\times10^3\,W$

Twist in the shaft, "θ" = 1°

Length of shaft, "L" = 2m = $2\times10^3\,mm$

no. of Revolutions, "N" = 200 rpm

shear stress, "τ" = $15\,N/mm^2$

modulus of rigidity, "G" = $1\times10^5\,N/mm^2$

$$P = \frac{2\pi NT}{60} \Rightarrow T = \frac{60P}{2\pi N} = \frac{60\times75\times10^3}{2\times\pi\times200}$$

$$= 3580.98\,N\text{-}m$$

$$= 3580.98\times10^3\,N\text{-}mm$$

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\frac{50}{D/2} = \frac{1\times10^{5}\times1^{\circ}\times\frac{\pi}{180}}{2000}$$

$$\frac{50\times2}{D} = 0.872$$

$$D = 114.67\,mm$$

(4) Two shafts of the same material and off same lengths are subjected to be same torque, if the 1st shaft is off a solid circular section and the second shaft is of hollow circular section, whose in-dia is 2/3 D and the max-shear stresses developed in the each shaft is same. find the external diameter.

Sol) Given data

$$d = \frac{2}{3}D$$

section ①   section ②

$L = L$

$\tau = \tau$

$T = T$

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow T = \tau\times\frac{J}{R}$$

$$T = \tau \cdot Z_p$$

Torque in solid shaft, $T_1 = \tau_1 \times Z_P = \tau_1 \times \frac{\pi d^3}{16} \rightarrow ①$

Torque in hollow shaft, $T_2 = \tau_2 \times Z_P = \tau_2 \times \frac{\pi}{16D}\left[D^4 - d^4\right] \rightarrow ②$

Equating ① & ②

$$T_1 = T_2$$

$$\cancel{\tau_1} \times \frac{\cancel{\pi} d^3}{\cancel{16}} = \cancel{\tau_2} \times \frac{\cancel{\pi}}{\cancel{16}D}\left[D^4 - d^4\right]$$

$$d^3 = \frac{D^4 - d^4}{D}$$

$$d^3 = \frac{1}{D}\left[D^4 - \left(\frac{2}{3}D\right)^4\right]$$

$$d^3 = \frac{D^{\cancel{4}}}{\cancel{D}}\left[1 - \frac{16}{81}\right]$$

$$d^3 = \frac{65}{81} D^3$$

$$\left[\frac{d}{D}\right]^3 = \frac{65}{81}$$

$$\left[\frac{2}{3}\right]^3 D^3 = \frac{65}{81}$$

$$\frac{8}{27} D^3 = \frac{65}{81}$$

$$D = \sqrt[3]{\frac{65 \times 27}{81 \times 8}} = 1.39\,mm$$

UNIT-5 COLUMNS

Columns : The vertical members of a building frame or any structural system carrying mainly axial compressive loads are called as columns.

Types of Columns

The columns can be divided into 3 classes based on slenderness ratio (λ) or length to diameter ratio (L/d).

(a) short columns $\rightarrow \lambda < 32$ or $\frac{L}{d} < 8$ or $L < 8d$

(b) long columns $\rightarrow \lambda > 120$ or $\frac{L}{d} > 30$

(c) medium columns $\rightarrow 32 < \lambda < 120$ (or) $32 \frac{L}{d} > 30$

a) short columns :

columns have length less than 8 times their respective diameter (or) slenderness ratio less than 32 are called short columns.

1) when short columns are subjected to compressive loads, buckling is negligible.

2) The short columns are always subjected to direct compressive stresses only.

b) medium columns :

The columns have their lengths varying from 8 times to 30 times their respective diameters or slenderness ratio lies b/w 32 to 120 are called intermediate or medium sized columns.

⇒ In the above columns both buckling and direct stresses are taken.

c) Long columns:

The columns have length more than 30 times their respective diameter or slenderness ratio more than 120 are called long columns.

⇒ The long columns are always subjected to buckling stress.

Euler's theory for axially loaded long columns:

To study the stability of long columns was made by Euler, he derived an equation for the buckling stress of long columns based on end conditions.

Assumptions:

1) The material of the column is homogenous and isotropic
2) The section of the column is uniform throughout.
3) The column is initially straight and is loaded axially
4) The column fails by buckling alone.
5) The self-weight of column is negligible.

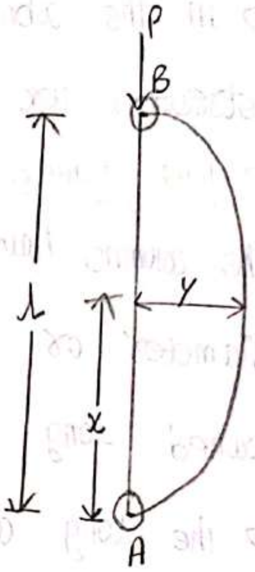
Types of end conditions:

In Euler's theory the following four types from end conditions.

1) columns with both ends hinged
2) columns with both ends fixed
3) columns with one end fixed and other end hinged

4) columns with one end fixed and other end free.

a) columns with Both ends hinged



1) consider the elastic long columns AB of length "l". If the both ends A & B are hinged. let "P" be the crippling load (or) buckling load (or) axial load at which the column has buckled.

2) Due to axial compressive load "P". let the deflecting "y" from "A" at a distance of "x".

Bending moment Due to axial load is given by

$$m = EI \frac{d^2y}{dx^2}$$

$$EI \cdot \frac{d^2y}{dx^2} = -Py$$

$$EI \frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \gamma y = 0 \longrightarrow ①$$

solution of above differential equation

$$y = C_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[x\sqrt{\frac{P}{EI}}\right] \rightarrow ②$$

C_1, C_2 are constant

Applying Boundary conditions

(i) At A ; $x=0$ & $y=0$

substitute in eq (2)

$$0 = C_1 \cos(0) + C_2 \sin(0)$$

$$= C_1 \times 1 + 0$$

$$C_1 = 0$$

(ii) At B; $x=l$ & $y=0$

substitute in eq (2)

$$0 = C_1 \cos\left[l\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[l\sqrt{\frac{P}{EI}}\right]$$

$$= 0 + C_2 \sin\left[l\sqrt{\frac{P}{EI}}\right]$$

$$C_2 \times \sin\left(l\sqrt{P/EI}\right) = 0$$

$$C_2 \neq 0;\ \sin\left(l\sqrt{P/EI}\right) = 0$$

$$l\sqrt{\frac{P}{EI}} = \sin^{-1}(0)$$

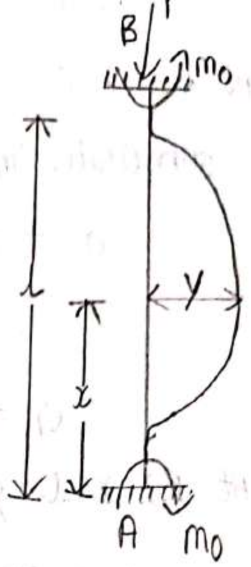
$$l\sqrt{P/EI} = \cancel{\sin^{-1}}(\cancel{\sin}\,\pi)$$

$$\sqrt{P/EI} = \frac{\pi}{l}$$

$$\frac{P}{EI} = \frac{\pi^2}{l^2}$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{l^2}}$$

b) Columns with Both ends fixed:



1) Consider the elastic long columns AB of length l. If the both ends are fixed. let "P" be the crippling load at which the column has buckled.

2) Due to axial load "P", let the deflection "y" from A at a distance of x.

3) Bending moment due to axial load is given by

$$EI\frac{d^2y}{dx^2} = m$$

$$EI\frac{d^2y}{dx^2} = m_0 - Py$$

$$EI\frac{d^2y}{dx^2} + Py = m_0$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{m_0}{EI} \rightarrow ①$$

The solution of above differential equation

$$y = C_1\cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2\sin\left[x\sqrt{\frac{P}{EI}}\right] + \frac{m_0}{P} \rightarrow ②$$

C_1 and C_2 are the constants.

Differentiate eq ②

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}}\sin\left[x\sqrt{\frac{P}{EI}}\right] + C_2\sqrt{\frac{P}{EI}}\cos\left[x\sqrt{\frac{P}{EI}}\right] + 0$$

$\rightarrow ③$

Boundary conditions:

(i) At A ; $x=0$; $y=0$ and $\frac{dy}{dx}=0$

substitute in eq ②

$$0 = C_1 \cos\left[0\sqrt{P/EI}\right] + C_2 \sin\left[0\sqrt{\frac{P}{EI}}\right] + \frac{m_0}{P}$$

$$0 = C_1 \times 1 + C_2(0) + \frac{m_0}{P}$$

$$C_1 = -\frac{m_0}{P}$$

substitute $x=0$, $\frac{dy}{dx}=0$ in eq ③

$$0 = -C_1\sqrt{\frac{P}{EI}}\sin\left[0\sqrt{\frac{P}{EI}}\right] + C_2\sqrt{\frac{P}{EI}}\cos\left[0\sqrt{\frac{P}{EI}}\right]$$

$$0 = -C_1(0) + C_2 \times \sqrt{\frac{P}{EI}}$$

$$C_2\sqrt{\frac{P}{EI}} = 0$$

$$C_2 = 0, \sqrt{\frac{P}{EI}} \neq 0$$

substitute above values in eq ②

$$y = -\frac{m_0}{P} \times \cos\left[x\sqrt{\frac{P}{EI}}\right] + \frac{m_0}{P} \rightarrow ④$$

(ii) At B; $x=l$, $y=0$

substitute in eq ④

$$0 = \frac{-m_0}{P}\cos\left[l\sqrt{\frac{P}{EI}}\right] + \frac{m_0}{P}$$

$$\frac{-\cancel{m_0}}{\cancel{P}} = \frac{-\cancel{m_0}}{\cancel{P}}\cos\left[l\sqrt{\frac{P}{EI}}\right]$$

$$1 = \cos\left[l\sqrt{P/EI}\right]$$

$$l\sqrt{\frac{P}{EI}} = \cos^{-1}(1)$$

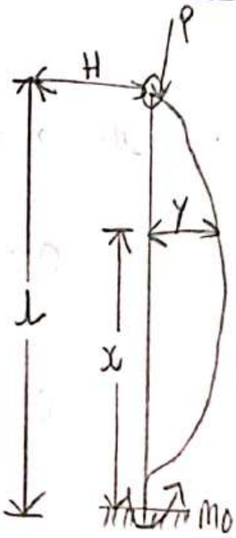
$$= \cos^{-1}(\cos 2\pi)$$

$$\frac{P}{EI} = \frac{4\pi^2}{l^2}$$

$$\boxed{P_{cr} = \frac{4\pi^2 EI}{l^2}}$$

c) columns with one end fixed and other end hinged

1) consider the elastic long column AB of length "l". If one end is fixed and other end is hinged. let "P" be the crippling load and H be the horizontal force at which the column has buckled.



2) Due to axial load P, let the deflection "y" from A at a distance of x.

3) Bending moment due to axial load is given by:

$$EI\frac{d^2y}{dx^2} = m$$

$$EI\frac{d^2y}{dx^2} = -Py + H(l-x)$$

$$EI\frac{d^2y}{dx^2} + Py = H(l-x)$$

$$\frac{d^2y}{dx^2} - \frac{P}{EI}y = \frac{H(l-x)}{EI} \rightarrow ①$$

solution of Above Differential equation.

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right) + \cancel{C_2\sqrt{\frac{P}{EI}}}\ \frac{H(l-x)}{P}$$

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}} \times \sin\left(x\sqrt{P/EI}\right) + C_2\sqrt{P/EI}\ \cos\left(x\ P/EI\right) - \frac{H}{P} \rightarrow ③$$

C_1 and C_2 constants.

Boundary conditions

(1) At A ; $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$

substitute $x = 0$; $\frac{dy}{dx} = 0$, eq ③

$$0 = -C_1\sqrt{\frac{P}{EI}}\ \sin(0) + C_2\sqrt{\frac{P}{EI}} \times \cos(0) - \frac{H}{P}$$

$$0 = -C_2\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$-C_2\sqrt{\frac{P}{EI}} = -\frac{H}{P}$$

$$C_2 = \frac{H}{P}\sqrt{\frac{P}{EI}}$$

substitute C_1 and C_2 in eq ②

$$y = \frac{-ml}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \times \sin\left(l\sqrt{\frac{P}{EI}}\right) + H\frac{(l-l)}{P}$$

$$\frac{Hl}{P} \times \frac{P}{H}\sqrt{\frac{P}{EI}} = \tan\left[l\sqrt{\frac{P}{EI}}\right]$$

$$\frac{Hl}{P}\cos\left(l\sqrt{P/EI}\right) = \frac{H}{P}\sqrt{\frac{EI}{P}}\ \sin\left[l\sqrt{\frac{P}{EI}}\right]$$

$$l\sqrt{\frac{P}{EI}} = \tan\left[l\sqrt{\frac{P}{EI}}\right]$$

The solution of above equation

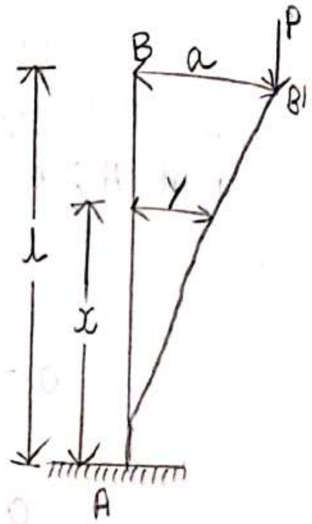
$$l\sqrt{\frac{P}{EI}} = 4.5 \text{ radians}$$

$$\frac{P}{EI} = \frac{4.5^2}{l^2}$$

$$P_{cr} = \frac{20.25^2 EI}{l^2} = \boxed{\frac{2\pi^2 EI}{l^2}}$$

d) Column with one end fixed and other end free:

1) Consider the elastic long column AB of length l. If one end is fixed and other end free and at which let "P" be the crippling load acting on the free end at which the column has buckled.



2) Due to axial load P, let the deflection "y" from "A" at a distance of x.

3) Bending moment due to axial load is given by

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$$EI \frac{d^2y}{dx^2} + Py = Pa$$

$$\frac{d^2y}{dx} + \frac{P}{EI} y = \frac{Pa}{EI} \rightarrow ①$$

The solution of above equation

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) + a \rightarrow ②$$

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}} \times \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}} \times \cos\left(x\sqrt{\frac{P}{EI}}\right)$$

Boundary conditions

At A; $x=0$, $y=0$ $\frac{dy}{dx}=0$

substitute in eq ②

$$0 = C_1 \cos(0) + C_2 \sin(0) + a$$

$0 = c_1 x_1 + a$

$c_1 = -a$

substitute $x = 0$, $\frac{dy}{dx} = 0$ in eq ③

$$0 = -c_1\sqrt{\frac{P}{EI}}\sin(0) + c_2\sqrt{\frac{P}{EI}}\cos(0)$$

$$c_2\sqrt{\frac{P}{EI}} = 0$$

$$c_2 = 0 \;;\; \sqrt{P/EI} = 0$$

substitute in eq ②

$$y = -a\cos\left(x\sqrt{P/EI}\right) + 0 + a \longrightarrow ④$$

(ii) At B; $x = l$, $y = a$

$$\cancel{a} = -\cancel{a}\cos\left(l\sqrt{P/EI}\right) + \cancel{a}$$

$$0 = -a\cos\left(l\sqrt{P/EI}\right)$$

$$\cos\left(l\sqrt{P/EI}\right) = 0$$

$$l\sqrt{P/EI} = \cos^{-1}(0)$$

$$l\sqrt{P/EI} = \cancel{\cos}^{-1}\left(\cancel{\cos}\,\pi/2\right)$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

$$P_{cr} = \boxed{\frac{\pi^2 EI}{4l^2}}$$ //

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Equivalent length or Effective length of column :

=> it is defined has the distance b/w two adjacent points of the column from the centre of the end points.

=> The crippling load for any type of end conditions is given by $\boxed{P_{cr} = \frac{\pi^2 EI}{Le^2}}$

Type of end condions	crippling load	Effective length
(i) Both ends hinged	$P_{cr} = \frac{\pi^2 EI}{l^2}$	$Le = l$
(ii) Both ends fixed	$P_{cr} = \frac{4\pi^2 EI}{l^2}$	$Le = l/2$
(iii) one end fixed & other end hinged	$P_{cr} = \frac{2\pi^2 EI}{l^2}$	$Le = l/\sqrt{2}$
(iv) one end fixed & other end free	$P_{cr} = \frac{\pi^2 EI}{4l^2}$	$Le = 2l$

05/10/2024

Euler's crippling stress

Radious of gyration

It is defined as the ratio of the square root of moment of inertia to the area of the section is is denoted by $K = \sqrt{\frac{I}{A}}$

$$I = AK^2$$

crippling load $P_{cr} = \frac{\pi^2 EI}{Le^2}$

$$= \frac{\pi^2 E \times AK^2}{Le^2}$$

$$P_{cr} = \frac{\pi^2 EA}{(Le/K)^2}$$

crippling stress "f_c" = $\frac{\text{crippling load}}{\text{area}}$

$$= \frac{\cancel{\pi^2 EA}/(Le/K)^2}{A}$$

$$= \frac{\pi^2 E\cancel{A}}{\cancel{A} \times \left(\frac{Le}{K}\right)^2} = \frac{\pi^2 E}{\left(\frac{Le}{K}\right)^2}$$

$$\lambda = \frac{Le}{K}$$

$$\boxed{F_{cr} = \frac{\pi^2 E}{\lambda^2}}$$

slenderness ratio :

it is defined as the ratio of the effective length to the radious of gyration it is denoted by "λ".

$$\boxed{\lambda = \frac{Le}{K}}$$

Limitations of Euler's Theory (2m)

Euler's formula for crippling stress $f_{cr} = \frac{\pi^2 E}{(Le/K)^2}$

for both ~~h~~ ends hinged ; $f_c = \frac{\pi^2 E}{(Le/K)^2} = \frac{\pi^2 E}{(L/K)^2}$

The slenderness ratio is increases the crippling stress decreases but column material the crippling stress can not be greater than the crushing stress in the limiting case the crippling stress is equal to crushing stress $f_{cr} = f_c$

$$\frac{\pi^2 E}{(Le/K)^2} = f_c$$

$$\frac{\pi^2 E}{f_c} = \left(\frac{L_e}{k}\right)^2 = \lambda^2$$

$$\frac{\pi^2 E}{f_c} = \lambda^2 \Rightarrow \sqrt{\frac{\pi^2 E}{f_c}} = \lambda$$

for mild steel, $f_c = 320 N/mm^2$

$E = 2 \times 10^5 N/mm^2$

$$\lambda = \sqrt{\frac{\pi^2 \times 2 \times 10^5}{320}} = 78.53$$

$\lambda > 120 \rightarrow$ long column – Euler's Theory

slenderness ratio is less than 120 for mild steel column for both end's hinged then Euler's will not be valid.

1) A solid round bar 3m long 5cm in diameter is used as a column with both end's hinged Determine the crippling load Take "E" = $2 \times 10^5 N/mm^2$.

sol) Given data

Length of the beam "l" = 3m = 3000mm

Diameter of the beam "d" = 5cm = 50mm

young's modulus "E" = $2 \times 10^5 N/mm^2$

Both end's hinged

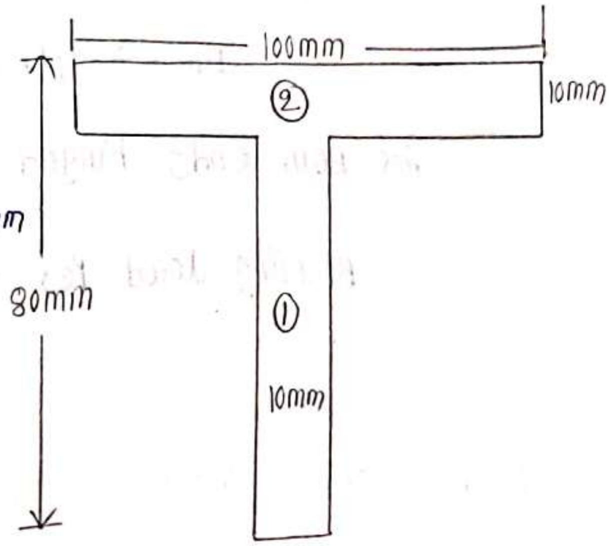
crippling load $P_{cr} = \frac{\pi^2 EI}{l^2}$

$$= \frac{\pi^2 \times 2 \times 10^5 \times \frac{\pi}{64} \times 50^4}{3000^2}$$

$= 67287.92N$

Determine the Buckling for a strut of T-section The flange width being 100mm and overall dearth 80mm and both flange and stem Thickness is 10mm. The strut is 3m long is hinged at both end's. Take $E = 200 GN/m^2$.

Given data



length of The strut "L" = 3m

$= 3000mm$

$$E = 200 \times 10^9 \frac{N}{10^6 N/mm^2}$$

$$= 200 \times 10^3 N/mm^2$$

$$= 2 \times 10^5 N/mm^2$$

$A_1 = 70 \times 10 = 700mm^2$

$A_2 = 100 \times 10 = 1000mm^2$

$y_1 = 70/2 = 35mm$

$y_2 = 70 + 10/2 = 75mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(700 \times 35) + (1000 \times 75)}{700 + 1000}$$

$$= 58.52mm$$

I_{xx} $$I_1 = \frac{b_1 d_1^3}{12} + A_1(\bar{y} - y_1)^2 = \frac{10 \times 70^3}{12} + 700(58.52 - 35)^2$$

$$= 673066.61mm^4$$

$$I_2 = \frac{b_2 d_2^3}{12} + A_2(\bar{y} - y_1)^2 = \frac{100 \times 10^3}{12} + 1000(58.52 - 75)^2$$

$$= 279923.73mm^4$$

$$I_{xx} = I_1 + I_2$$

$$= (673066.61 + 279923.73)$$

$$= 952990.34mm^4$$

$$I_{YY} = \frac{d_1 b_1^3}{12} + \frac{d_2 b_2^3}{12} = \frac{70 \times 10^3}{12} + \frac{10 \times 100^3}{12}$$

$$= 839166.66 \, mm^4$$

so, "I" least value is "I_{yy}"

$$I_{yy} = 839166.66 \, mm^4$$

for Both ends hinged

$$\text{Buckling load "Pcr"} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2 \times 10^5 \times 839166.66}{3000^2}$$

$$= 184049.84 \, N$$

3) A column of timber sections 15cm x 20cm is 6m long both ends beam fixed. if the young's modulus for their 17.5 kN/mm². Determine (i) crippling load and (ii) safe load for the column if factor of safety is "3".

sol) Given data

Dimensions of section

breadth "b" = 15cm = 150mm

depth "d" = 20cm = 200mm

length of the section "l" = 6m = 6000mm

Young's modulus "E" = 17.5 kN/m²

$= 17.5 \times 10^3 \, N/mm^2$

$$\text{Effective length } (Le) = l/2 = 6000/2$$

$= 3000mm$

Rectangular section "I" = $\frac{bd^3}{12} = \frac{150 \times 200^3}{12} \Rightarrow 100 \times 10^6 \, mm^4$

(i) crippling load

$$P_{cr} = \frac{4\pi^2 EI}{l^2} = \frac{4 \times \pi^2 \times 17.5 \times 10^3 \times 100 \times 10^6}{6000^2}$$

$$= 1.919 \times 10^6 N.$$

$$= 1919 \times 10^3 N$$

(ii) safe load

$$= \frac{\text{crippling load}}{F.O.S} = \frac{1919 \times 10^3}{3}$$

$$= 639 \times 10^3 N$$

$$= 639 \, kN$$

4) a hollo mild steel tube 6m long 4cm internal diameter and 5mm thickness is used as a column with both end's hinged. find the crippling load and safe load factor of safety is "3". Take $E = 2 \times 10^5 \, N/mm^2$

A) Given data

Length of the tube "L" = 6m = 6000mm

Internal diameter "d" = 4cm = 40mm

External diameter "D" = d + 2T

$$= 40 + 2 \times 5 = 50mm$$

factor of safety = 3

moment of inertia "I" = $\frac{\pi}{64}(D^4 - d^4)$

$$= \frac{\pi}{64}(50^4 - 40^4)$$

$$= 181132.45 mm^4 \Rightarrow 181.13 \times 10^3 mm^4$$

Both ends hinged

(i) crippling load

$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2\times10^5 \times 181.13\times10^3}{6000^2}$$

$$= 9931.56\,N$$

$$= 9.931\,kN$$

(ii) safe load

$$\frac{\text{crippling load}}{F.O.S} = \frac{9.931}{3}$$

$$= 3.310\,kN$$

Rankine's Theory:

On the basis of results of exetements performed by Rankine the established an empherical formula is applicable to all type of columns. The empherical formula given by rankine is known as rankine's formula is $\frac{1}{P} = \frac{1}{P_C} + \frac{1}{P_E}$

P = critical load by Rankine formula

P_C = crushing load $\Rightarrow \sigma_c \times A$

P_E = critical load by Euler's formula

$$\frac{1}{P} = \frac{P_E + P_C}{P_C \times P_E}$$

$$P = \frac{P_C \times P_E}{P_E + P_C} \Rightarrow \frac{P_C}{\frac{\cancel{P_E}}{\cancel{P_E}} + \frac{P_C}{P_E}}$$

$$P = \frac{P_C}{1+\frac{P_C}{P_E}}$$

$$P = \frac{\sigma_C \times A}{1+(\sigma_C \times A) \times \frac{Le}{\pi^2 EI}}$$

$$P = \frac{\sigma_C \times A}{1+\frac{\sigma_C A Le^2}{\pi^2 EI}}$$

$$P = \frac{\sigma_C \times A}{1+\frac{\sigma_C \cancel{A} Le^2}{\pi^2 E \cancel{A} k^2}}$$

$$P = \frac{\sigma_C \times A}{1+\frac{\sigma_C}{\pi^2 E} \times \left(\frac{Le}{k}\right)^2}$$

$$\boxed{P = \frac{\sigma_C \times A}{1+a \times \lambda^2}}$$

Rankine's constant $a = \frac{\sigma_C}{\pi^2 E}$

Determine the section of CI, a hallow symmetrical column 3m long with both ends firmly built in. If it carries an axial load 800kN. The ratio of internal diameter is $\frac{5}{8}$. Use factor of safety $=4$. Take $\sigma_C = 550\,N/mm^2$ and rankine's constant $a = 1/1600$.

Given data

crushing stress "σ_C" $= 550\,N/mm^2$

length of column "l" $= 3m = 3000mm$

Axial load "P" $= 800kN = 800 \times 10^3 N$

Rankine's constant "a" $= 1/1600$

factor of safety = 4

$\frac{d}{D} = \frac{5}{8} \Rightarrow d = \frac{5}{8}D$

Both ends fixed,

Effective length "Le" $= l/2 = \frac{3000}{2} = 1500\,mm$

Area of cylindrical section;

$$A = \frac{\pi}{4}\left[D^2 - d^2\right] = \frac{\pi}{4}\left[D^2 - \left[\frac{5}{8}D\right]^2\right]$$

$$= \frac{\pi D^2}{4}\left[1 - \frac{25}{64}\right] = 0.478D^2$$

moment of inertia;

$$I = \frac{\pi}{64}\left[D^4 - d^4\right] = \frac{\pi}{64}\left[D^4 - \left[\frac{5}{8}D\right]^4\right]$$

$$= \frac{\pi D^4}{64}\left[1 - \frac{625}{4096}\right]$$

$$= 0.0415D^4$$

Radious of gyration; $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.0415D^4}{0.478D^2}}$

$$= 0.294D.$$

critical load = safe load x F.O.S

$$= 8\times10^5 \times 4 = 32\times10^5\,N$$

substitute the above values in Rankine's equation.

$$P_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{Le}{K}\right)^2}$$

$$32\times10^5 = \frac{550\times0.478D^2}{1+\frac{1}{1600}\left[\frac{1500}{0.294D}\right]^2}$$

$$D = 146.34mm$$

2) Hollow cast Iron whose external Diameter is 200mm and has the Thickness of 20mm 4.5 long and is fixed at both ends calculate the safe load by rankine's formula using factor of safety 2.5. find the ratio of Euler's to rankine's load. Take $E=1\times10^5$ N/mm² and $a = 1/1600$, $\sigma_c = 550$ N/mm².

A) Given data

External Diameter "D" = 200mm

Thickness 'T' = 20mm

Internal Diameter "d" = 200-2t = 200 - 2×20 = 160mm

length of column "l" = 4.5m = 4500mm

young's modulus "E" = 1×10^5 N/mm²

crushing stress "σ_c" = 550 N/mm²

factor of safety = 2.5

"a" = 1/1600

$$I = \frac{\pi}{64}(D^4-d^4) = \frac{\pi}{64}\left[200^4-160^4\right]$$

$$= 46.36\times10^6\ mm^4$$

Euler's crippling load,

$$P_{cr} = \frac{4\pi^2 EI}{l^2} = \frac{4\times\pi^2\times 1\times10^5\times 46.36\times10^6}{(4500)^2}$$

$$= 9.038\times10^6 kN$$

$$= 9038\times10^3 N$$

$$\text{Area "A"} = \frac{\pi}{4}\left[D^2-d^2\right] = \frac{\pi}{4}\left[200^2-160^2\right]$$

$$= 11.30\times10^3 mm^2$$

Radious of gyration;

$$K=\sqrt{\frac{I}{A}} = \sqrt{\frac{46.36\times10^6}{11.30\times10^3}} = 64.05 mm$$

Effective length, $Le = l/2 = \frac{4500}{2} = 2250 mm$

Rankine's crippling load;

$$P_{cr} = \frac{\sigma_c\times A}{1+a\left(\frac{le}{k}\right)^2} = \frac{550\times11.30\times10^3}{1+\frac{1}{1600}\left[\frac{2250}{64.05}\right]^2}$$

$$= 3.508\times10^6 N$$

$$= 3508\times10^3 N$$

$$\text{safe load} = \frac{\text{Rankine's } P_{cr}}{F.O.S} = \frac{3508\times10^3}{2.5}$$

$$= 1403200 N$$

$$\text{Ratio} = \frac{P_E}{P_{cr}} = \frac{9038\times10^3}{3508\times10^3}$$

$$= 2.57$$

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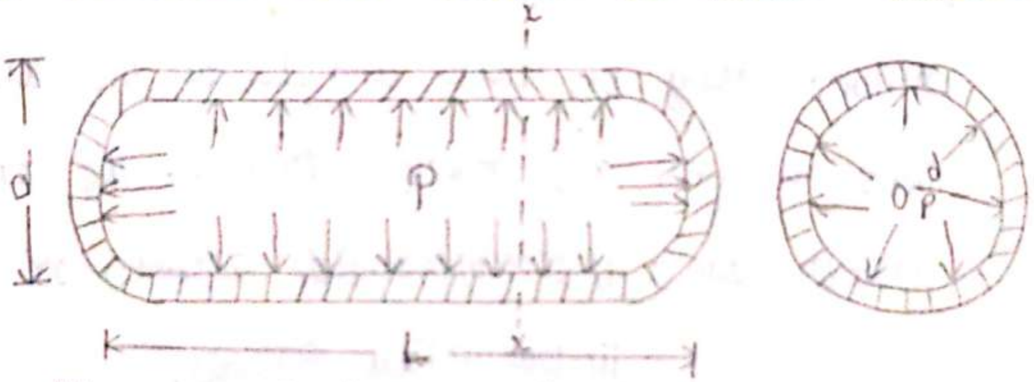
Part → 2 Thin cylinders

Thin cylinder : The thickness of the wall of the cylindrical vessel is less than 1/15 to 1/20 of internal diameter. that cylindrical vessel is known as thin cylinder.

→ These cylinders are used for storing fluids under pressure.

→ In thin cylinders, the stress distribution is uniform over the thickness of the wall.

Thin cylindrical vessel subjected to internal pressure:



Let "d" = Internal diameter of thin cylinder

"t" = thickness of cylinder

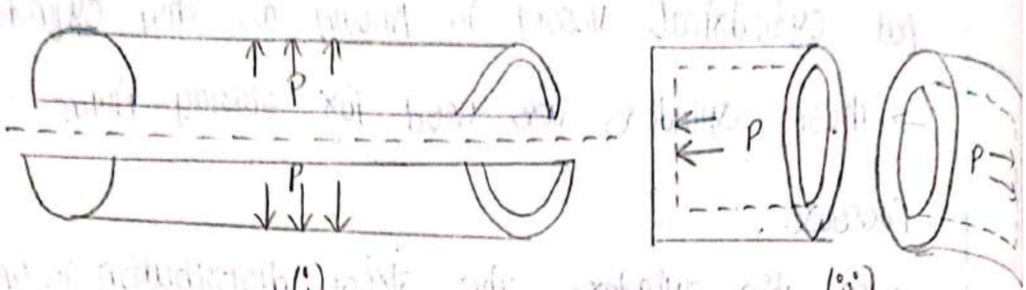
"p" = Internal pressure of fluid.

"L" = length of the cylinder

The cylinder fail by splitting up in two ways because of internal pressure.

⇒ The forces due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, it will burst along the axis length of the cylinder (as shown in fig(i)).

⇒ The forces due to pressure of the fluid acting at the ends of the thin cylinder it will burst along the perpendicular to the axis as shown in Fig (ii)



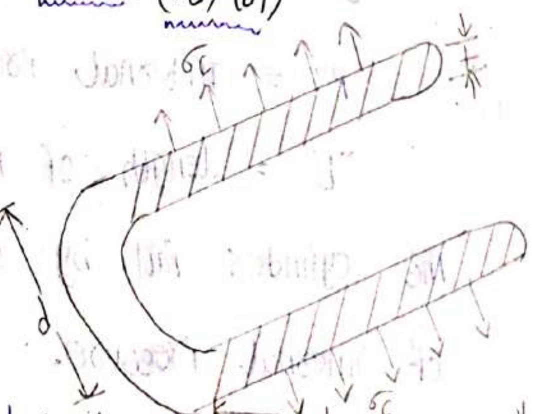
(i) (ii)

When a thin cylinders is subjected to internal fluid pressure, the stresses will be developed in the cylinder along the axis and ⊥ to the axis. These stresses are tensile stresses. The tensile stresses are : i) circumferential stresses or Hoop stress
ii) longitudinal stresses.

14/10/2024

Expression for circumferential stress (σ_c) (σ_1)

Consider a thin cylinder subjected to an internal fluid pressure the circum--ferential stress will be developed in the thin cylinder due to fluid pressure.



Let "L" = length of the cylinder
"P" = Internal fluid pressure
"D" = Diameter of the cylinder
"t" = Thickness of the thin cylinder

force due to fluid pressure

$$= P \times (L \times d) \rightarrow ①$$

force due to circumferential stress

$$= \sigma_c \times (L \times t) \times 2 \rightarrow ②$$

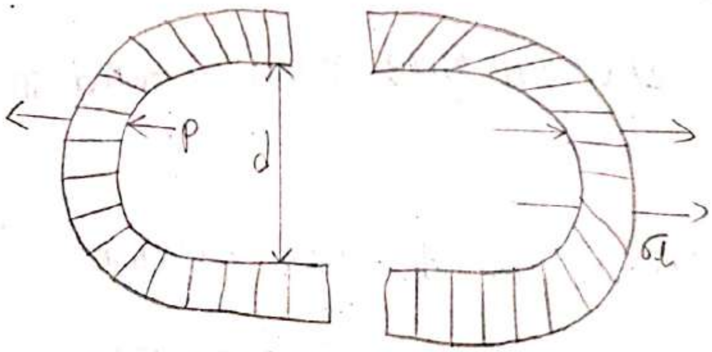
equating eq ① & ②

$$P \times (\cancel{L} \times d) = \sigma_c \times (\cancel{L} \times t) \times 2$$

$$\boxed{\sigma_c = \frac{Pd}{2t}}$$

Expression for longitudinal stress (σ_l) (σ_2)

consider a thin cylinder subjected to internal fluid pressure the longitudinal stress will be developed in the cylinder. If brushting the cylinder takes places along the section. In that cross forces sxued be developed the cylinder.



force due to fluid pressure

$$= P \times \frac{\pi}{4} d^2 \rightarrow ①$$

force due to longitudinal stress

$$= \sigma_l \times \pi d t \rightarrow ②$$

equating eq ① & ②

$$P \times \frac{\pi}{4} d^2 = \sigma_l \times \pi d t \qquad \boxed{\sigma_l = \frac{Pd}{4t}}$$

note: $\sigma_c = \frac{1}{2}\left(\frac{Pd}{2t}\right)$

$\sigma_L = \frac{1}{2}\sigma_c$

longitudinal stress is $\frac{1}{2}$ of the circumferential stress.

maximum shear stress $\tau = \frac{\sigma_1 - \sigma_2}{2}$

$$\tau = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{\sigma_c - \sigma_L}{2}$$

$$= \frac{Pd/2t - Pd/4t}{2} = \frac{2Pd - Pd}{8t} = \boxed{\frac{Pd}{8t}}$$

1) A cylindrical pipe of diameter 1.5m and thickness 1.5cm subjected to an internal fluid pressure of 1.2N/mm². Determine 1) longitudinal stress developed in the pipe

2) circumference stress developed in the pipe

sol) Given data

Diameter of the pipe "d" = 1.5m = 1500mm

thickness of the pipe "t" = 1.5cm = 15mm

Internal fluid pressure "P" = 1.2N/mm²

i) longitudinal stress developed in the pipe

$$\sigma_L = \frac{Pd}{4t} = \frac{1.2 \times 1500}{4 \times 15}$$

$$= 30\,N/mm^2$$

ii) circumference stress $\sigma_c = \frac{Pd}{2t} = \frac{1.2 \times 1500}{2 \times 15}$

$= 60 N/mm^2$.

2) A cylinder of Internal diameter 1.25m consist a fluid at an internal pressure of $2N/mm^2$. Determine maximum thickness of the cylinder. If the

i) the longitudinal stress is not to exceed $30N/mm^2$

ii) the circumferential stress is not to exceed $45N/mm^2$.

sol) Given data

Diameter of the pipe "d" = 1.25m = 1250mm

Internal pressure "P" = $2N/mm^2$

longitudinal stress $\sigma_l = 30N/mm^2$

$$\sigma_l = \frac{Pd}{4t}$$

$$30 = \frac{2 \times 1250}{4 \times t} \quad t = 20.83mm$$

circumference stress $\sigma_c = 45N/mm^2$

$$\sigma_c = \frac{Pd}{2t}$$

$$45 = \frac{2 \times 1250}{2 \times t} \quad t = 27.77mm$$

maximum thickness of cylinder "t_c" = 27.77mm

Change in Diameter, Length and Volume (10m) 15/10/2024

For thin cylinder subjected to internal fluid pressure (P) developing the circumferential stress (σ_c) and longitudinal stress (σ_l).

circumferential stress "σ_c" $= \frac{Pd}{2t}$

longitudinal stress "σ_l" $= \frac{Pd}{4t}$

let "μ" = Poisson's ratio

"δd" = change in Diameter

"δL" = change in length

"δV" = change in volume

"e_c" = circumferential strain

"e_l" = longitudinal strain

(i) circumferential strain $e_c = \frac{\sigma_c}{E} - \mu \frac{\sigma_l}{E}$

$$= \frac{Pd/2t}{E} - \mu \frac{Pd/4t}{E}$$

$$= \frac{Pd}{2tE} - \mu \frac{Pd}{4tE}$$

$$e_c = \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right) \rightarrow ①$$

circumferential strain $e_c = \frac{\delta d}{d} \rightarrow ②$

equating eq ① & ②

$$\frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right) = \frac{\delta d}{d}$$

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)} \rightarrow \text{formula}$$

ii) longitudinal strain, $e_l = \frac{\sigma_l}{E} - \mu \frac{\sigma_c}{E}$

$$= \frac{Pd}{4tE} - \mu \frac{Pd}{2tE}$$

$$e_l = \frac{Pd}{4tE}(1-2\mu) \rightarrow ③$$

longitudinal strain $e_l = \frac{\delta l}{l} \rightarrow ④$

equating eq ③ & ④

$$\frac{Pd}{4tE}(1-2\mu) = \frac{\delta l}{l}$$

$$\boxed{\delta l = \frac{Pdl}{4tE}(1-2\mu)} \rightarrow \text{formula}$$

iii) volumetric strain, $e_v = \frac{\delta v}{V}$

$$= \frac{\text{Final volume} - \text{original volume}}{\text{original volume}}$$

original volume, $V_o = A \times L$

$$= \frac{\pi}{4} d^2 \times L \rightarrow ⑤$$

Final volume, $V_f = A \times L$

$$= \frac{\pi}{4}(d+\delta d)^2 \times (L+\delta L)$$

$$= \frac{\pi}{4}(d^2+2d\,\delta d+\delta d^2)(L+\delta L)$$

$$V_f = \frac{\pi}{4}\left(d^2L + 2d\,\delta d L + \delta d^2 L + d^2 \delta L + 2d\,\delta d\,\delta l + \delta d^2 \delta l\right)$$

neglecting the small terms

$$V_f = \frac{\pi}{4}\left(d^2 l + 2d\,\delta d\, l + d^2 \delta l\right)$$

volumetric strain, $e_v = \frac{\frac{\pi}{4}(d^2L + 2d\delta dL + d^2\delta L) - \frac{\pi}{4}d^2 \times L}{\frac{\pi}{4} \times d^2 L}$

$$= \frac{\cancel{\frac{\pi}{4}d^2L} + \frac{\pi}{4}2d\delta dL + \frac{\pi}{4}d^2\delta L - \cancel{\frac{\pi}{4}d^2L}}{\frac{\pi}{4} \times d^2 L}$$

$$= \frac{\cancel{\frac{\pi}{4}}\, 2d\delta dL + d^2\delta L}{\cancel{\frac{\pi}{4}}\, d^2 L}$$

$$= \frac{2\cancel{d}\,\delta d\cancel{L}}{d^2 \cancel{L}} + \frac{\cancel{d^2}\,\delta L}{\cancel{d^2}L}$$

$$= 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_c + e_l$$

$$e_v = 2e_c + e_l$$

$$= \cancel{2} \times \left[\frac{Pd}{\cancel{2}tE}\left(1 - \mu/2\right) + \frac{Pd}{4tE}(1 - 2\mu)\right]$$

$$= \frac{Pd}{tE}\left(1 - \frac{\mu}{2}\right) + \frac{Pd}{4tE}(1 - 2\mu)$$

$$= \frac{Pd}{tE}\left(1 - \frac{\mu}{2} + \frac{1}{4} + \frac{2\mu}{4}\right)$$

$$e_v = \frac{Pd}{tE}\left(\frac{5}{4} - \mu\right) \rightarrow 7$$

volumetric strain $e_v = \frac{\delta V}{V} \rightarrow 8$

equating eq 7 & 8

$$\frac{\delta V}{V} = \frac{Pd}{tE}\left(\frac{5}{4} - \mu\right)$$

$$\boxed{\delta V = \frac{PdV}{tE}\left(\frac{5}{4} - \mu\right)} \rightarrow \text{formula}$$

1) calculate the change in Diameter, change in length and volume in thin cylinder 100cm dia. 1cm thickness and 5m long when subjected to a internal pressure $3 N/mm^2$. Take the value of $E = 2\times10^5 N/mm^2$ and Poisson's ratio 0.3.

Sol) Given data

diameter of the cylinder, "d" = 100cm = 1000mm

Thickness of the cylinder "t" = 1cm = 10mm

length of the cylinder "l" = 5m = 5000mm

Internal pressure "p" = $3 N/mm^2$

young's modulus "E" = $2\times10^5 N/mm^2$

Poisson's ratio "μ" = 0.3

i) change in Diameter, $\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$

$$= \frac{3\times1000^2}{2\times10\times2\times10^5}\left(1-\frac{0.3}{2}\right)$$

$= 0.673mm$

ii) change in length, $\delta l = \frac{pdL}{4tE}(1-2\mu)$

$$= \frac{3\times1000\times5000}{4\times10\times2\times10^5}(1-2\times0.3)$$

$= 0.75mm$

iii) change in volume, $\delta v = \frac{pdV}{tE}\left(\frac{5}{4}-\mu\right)$

$$= \frac{3\times1000\times\frac{\pi}{4}\times1000^2\times5000}{10\times2\times10^5}\left(\frac{5}{4}-0.\right.$$

$= 5.59\times10^6 mm^3$

8) A cylindrical shell 90cm long, 20cm internal diameter having thickness of metal have 8mm is filled with fluid of atmospheric pressure additional $20cm^3$ of fluid pumped into the cylinder find 1) pressure exisled by the fluid. 2) the hoop stress (or) circumferential stress.
$E = 2\times10^5 N/mm^2$ and $\mu = 0.3$.

Sol) Given data

Length of the cylinder "L" $= 90cm = 900mm$

Internal diameter "d" $= 20cm = 200mm$

Thickness of the cylindrical shell "t" $= 8mm$

Young's modulus "E" $= 2\times10^5 N/mm^2$

Possion's ration "μ" $= 0.3$

change in volume, $\delta V = 20cm^3 = 20\times10^3 mm^3$

i) volumetric strain, $e_v = \frac{Pd}{tE}\left(\frac{5}{4}-\mu\right)$

$$\frac{\delta V}{V} = \frac{Pd}{tE}\left(\frac{5}{4}-\mu\right)$$

$$\delta V = \frac{PdV}{tE}\left(\frac{5}{4}-\mu\right)$$

$$20\times10^3 = \frac{P\times200\times\frac{\pi}{4}\times200^2\times900}{8\times2\times10^5}\left(\frac{5}{4}-0.3\right)$$

$$P = 5.956 N/mm^2$$

ii) circumferential stress, $\sigma_c = \frac{Pd}{2t}$

$$= \frac{5.956\times200}{2\times8}$$

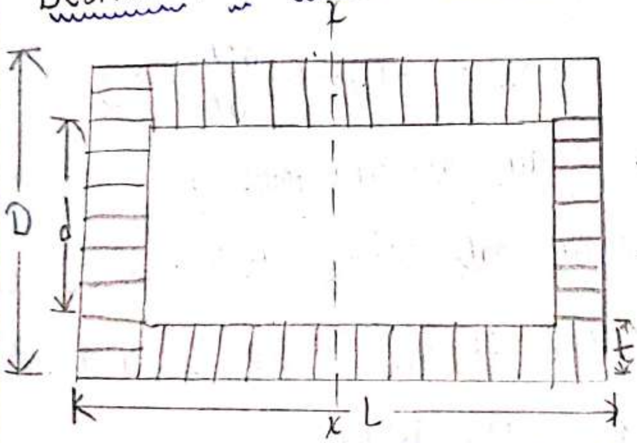
$$\sigma_c = 74.45 N/mm^2$$

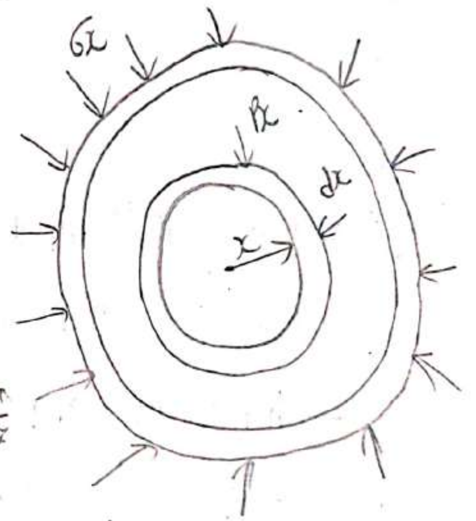
Part – ③ Thick cylinders

If the ratio of the thickness to internal diameter $\left(\frac{t}{d} > \frac{1}{20}\right)$ it is known as thick cylinders.

=> The hoop stress in the thick cylinder will not be uniform across the thickness actually the hoop stress will carry from a maximum value at the in the circumferential to a minimum value at the outer circumference.

Derivation of Lame's equation





Let, "R" = external radious of the cylinder

"r" = internal radious of the cylinder

=> consider an elementary Rim of the cylinder of radious "x" and thickness "dx"

=> let P_x = Radial pressure on the inner surface of the rim.

$P_x + dP_x$ = Radial pressure on the outer surface of the rim.

=> Take section x-x

force due to radial pressure,

$= P_x \times 2(x \times L) - (P_x + dP_x) \times 2(x + dx) L$

$$= 2P_x \times x \times L - 2(P_x \, dx \, L + \ldots$$

$$= \cancel{2P_x x L} - \cancel{2P_x x L} - 2P_x \, dx \, L - 2\cancel{P_x dx} \, dP_x L - 2 dP_x \ldots$$

$$= -2(P_x \, dx \, L + dP_x \times x \times L) \rightarrow (1)$$

force due to hoop stress $= \sigma_x \times 2 \times dx \times L \rightarrow (2)$

equating eq (1) & (2)

$$-\cancel{2}(P_x \, dx \, L + dP_x \times x \times L) = \sigma_x \times \cancel{2} \times dx \times L$$

$$-\left(\frac{P_x \, dx}{dx} + \frac{dP_x \times x}{dx}\right) = \sigma_x$$

$$\sigma_x = -P_x - x \frac{dP_x}{dx} \rightarrow (3)$$

$\Rightarrow$ The longitudinal strain at any point in the section is constant hence longitudinal stress will also be constan

$\Rightarrow$ hence at any point at a distance "x" from the centre. Three stress are acting.

i) radial compressive stress, P_x

ii) circumferential stress, σ_c

iii) longitudinal stress, σ_l.

The longitudinal strain at any point

$$e_l = \frac{\sigma_l}{E} - \mu \frac{\sigma_c}{E} + \mu \frac{P_x}{E}$$

longitudinal strain e_l is constant

$$\frac{\sigma_l}{E} - \mu \frac{\sigma_c}{E} + \mu \frac{P_x}{E} = \text{constant}$$

$$-\frac{\mu}{E}\left(\sigma_c - P_x\right) = \text{constant}$$

$$\sigma_c - P_x = \text{constant}$$

$$\sigma_x = P_x + 2a \rightarrow (4)$$

from (3) and (4)

$$-P_x - x\frac{dP_x}{dx} = P_x + 2a$$

$$-x\frac{dP_x}{dx} = P_x + 2a + P_x$$

$$x\frac{dP_x}{dx} = -2P_x - 2a$$

$$\frac{dP_x}{dx} = \frac{-2(P_x+a)}{x}$$

$$\frac{dP_x}{(P_x+a)} = -2\frac{dx}{x}$$

Integrating the above equation

$$\int \frac{dP_x}{(P_x+a)} = -2\int \frac{dx}{x}$$

$$\log_e(P_x+a) = -2\log_e x + \log_e b$$

$$\log_e(P_x+a) = -\log_e x^2 + \log_e b$$

$$= \log_e b - \log_e x^2$$

$$\log_e(P_x+a) = \log_e(b/x^2)$$

$$P_x + a = b/x^2$$

$$P_x + a = b/x^2$$

$$\boxed{P_x = \frac{b}{x^2} - a} \rightarrow (5)$$

Radial pressure equation

substitute P_x in eq(1)

$$\sigma_x = P_x + 2a$$

$$= \frac{b}{x^2} - a + 2a$$

$$\boxed{\sigma_x = \frac{b}{x^2} + a} \rightarrow \text{hoop stress equation}$$

The radial stress $P_x = \frac{b}{x^2} - a$, hoop stress $\sigma_x = \frac{b}{x^2}$

This two equations are called lem's equations The constants "a" and "b" are obtained from boundary condition's.

i) $x = r$, $P_x = P$

ii) $x = r_2$, $P_{r_2} = 0$

Problems

1) Determine the maximum and minimum across the section of the pipe of 400mm internal Diameter and 100mm Thickness when the pipe contains a fluid at a pressure 8 N/mm².

Sol) Given data

Internal Diameter "d" = 400mm

Internal radius "r" = $\frac{400}{2}$ = 200mm

External Diameter "D" = d+2t 400+(2×100) = 600mm

External radius "R" = $\frac{600}{2}$ = 300mm

Internal fluid pressure "P" = 8N/mm²

The radial pressure $= p_x = \frac{b}{x^2} - a \rightarrow ①$

By using Boundary conditions:

i) $x = r = 200mm, \quad p_x = P = 8N/mm^2$

ii) $x = R = 300mm, \quad p_x = P = 0$

substitute Boundary conditions in eq ①

$$① \Rightarrow 8 = \frac{b}{200^2} - a \rightarrow ②$$

$$② \Rightarrow 0 = \frac{b}{300^2} - a$$

$$a = \frac{b}{300^2} \rightarrow ③$$

substitute "a" value in eq ②

$$8 = \left[\frac{b}{200^2} - \frac{b}{300^2}\right]$$

$$8 = b\left[\frac{1}{200^2} - \frac{1}{300^2}\right]$$

$$\text{"b"} = 576000$$

substitute "b" value in eq ③

$$a = \frac{576000}{300^2}$$

$$= 6.4$$

The hoop stress is $\sigma_x = \frac{b}{x^2} + a$

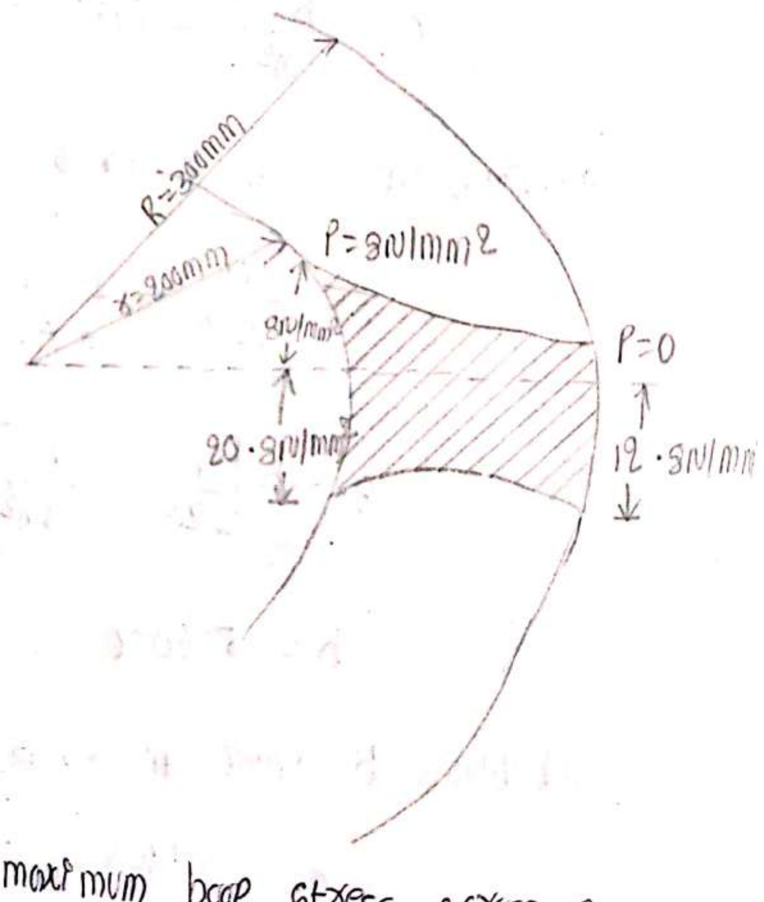
maximum hoop stress at "x" = r = 200mm

$$\sigma_{200} = \frac{576000}{200^2} + 6.4 = 20.8 \, N/mm^2$$

minimum hoop stress at "x" = R = 300mm

$$\sigma_{300} = \frac{576000}{300^2} + 6.4 = 12.8 \, N/mm^2$$

Radial and hoop stress Distrubution Diagram:



2) Determine the maximum hoop stress across the section of the pipe. External Diameter 200mm and internal Diameter is 100mm when the pipe is subjected to as internal fluid pressure 12.5N/mm². also sketch Radial pressure Distribution and hoop stress Distribution.

5Q1) Given data

External Diameter "D" = 200mm

External Radius "R" = $\frac{200}{2}$ = 100mm

Internal Diameter "d" = 100mm

Internal radius "r" = $\frac{100}{2}$ = 50mm

Internal fluid pressure "P" = 12.5 N/mm²

Radial pressure, $P_x = \frac{b}{x^2} - a \rightarrow ①$

Using Boundary conditions :

i) $x = r = 50mm$; $P_x = P = 12.5 N/mm^2$

ii) $x = R = 100mm$; $P_x = 0$

substitute in eq ①

$$① \Rightarrow 12.5 = \frac{b}{50^2} - a \rightarrow ②$$

$$② \Rightarrow 0 = \frac{b}{100^2} - a$$

$$a = \frac{b}{100^2} \rightarrow ③$$

Substitute "a" value in eq ②

$$12.5 = \left(\frac{b}{50^2} - \frac{b}{100^2}\right)$$

$$12.5 = b\left[\frac{1}{50^2} - \frac{1}{100^2}\right] \Rightarrow b = 41666.6$$

substitute "b" value in eq ③

$$a = \frac{41666.6}{100^2} = 4.16$$

maximum hoop stress at $x = r = 50$ mm

$$\sigma_{50} = \frac{b}{x^2} + a$$

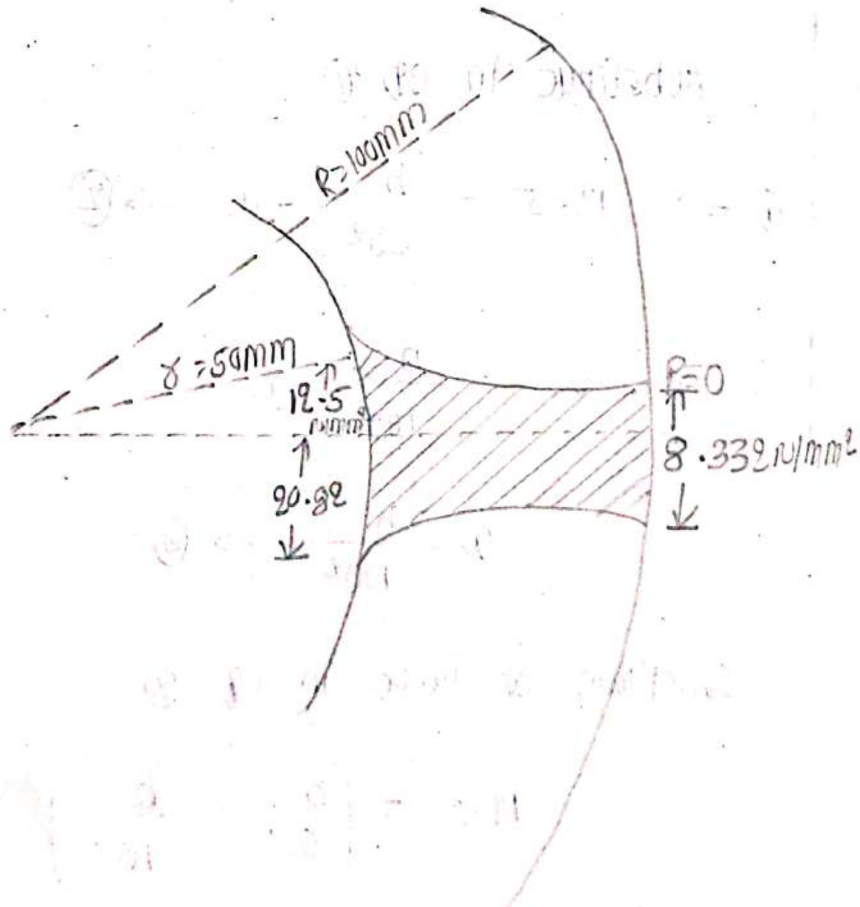
$$\sigma_{50} = \frac{41666.6}{50^2} + 4.16$$

$$= 20.82 \text{ N/mm}^2$$

minimum hoop stress at $x = r = 100$ mm

$$\sigma_{100} = \frac{41666.6}{100^2} + 4.16$$

$$= 8.322 \text{ N/mm}^2$$



Radial and hoop stress Distribution Diagram:

Compound Thick Cylinders:

In the compound cylinder is subjected to internal fluid pressure both the inner and outer cylinders will be subjected to hoop tensile stress compound thick cylinders made up of two cylinders that outer cylinder and inner cylinder.

Let r = Inner radious of the compound cylinder

r^* = Radious of the Junction of compound cylinder

R = outer radious of the compound cylinder

p^* = Radial pressure at the Junction

The net effective of the hoop stress or resultant stress will be the algebraic sum of the intial stress due to shrinking and due to internal fluid pressure.

1) The compound cylinder is made by shrinking a cylinder of external Diameter 300mm and internal Diameter 250mm over. The another cylinder of external diameter 250mm and internal Diameter 200mm, The radial pressure at the Junction 8 N/mm². find The final stress set up across the section when the compound

cylinder is subjected to an internal fluid pressure of $84.5 N/mm^2$.

sol) Given data

For outer cylinder:

External Diameter "D" = 300mm

External Radius "R" = $\frac{300}{2}$ = 150mm

Internal Diameter "d" = 250mm

Internal radius "r" = $\frac{250}{2}$ = 125mm

for inner cylinder:

External Diameter, "D" = 250mm

External radius "R" = $\frac{250}{2}$ = 125mm

Internal diameter "d" = 200mm

Internal radius "r" = $\frac{200}{2}$ = 100mm

pressure at junction. P^* = $8 N/mm^2$

Internal fluid pressure, $P = 84.5 N/mm^2$.

i) stress due to shrinking in the outer and inner cylinder before admiting the fluid pressure.

a) outer cylinder

The lame's equation for outer cylinder

$$P_x = \frac{b_1}{x^2} - a_1 \rightarrow (1)$$

$$\sigma_x = \frac{b_1}{x^2} + a_1 \rightarrow (2)$$

Using boundary conditions

i) $x_1 = r = 125mm$; $P_x = P = 8N/mm^2$

ii) $x_1 = R = 150mm$; $P_x = 0$

substitute boundary conditions in eq (1)

$$(1) \Rightarrow \quad 8 = \frac{b_1}{125^2} - a_1 \rightarrow (3)$$

$$(2) \Rightarrow \quad 0 = \frac{b_1}{150^2} - a_1$$

$$a_1 = \frac{b_1}{150^2} \rightarrow (4)$$

substitute 'a_1' value in eq (3)

$$8 = \left[\frac{b_1}{125^2} - \frac{b_1}{150^2}\right]$$

$$8 = b_1\left[\frac{1}{125^2} - \frac{1}{150^2}\right]$$

$$b_1 = 409090.90$$

substitute "b_1" value in eq (4)

$$a_1 = \frac{409090.90}{150^2}$$

$$= 18.18$$

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substitute "a_1" "b_1" values in eq ②

The hoop stress in the outer cylinder, $\sigma_x = \frac{b_1}{x^2} + a_1$

$$\sigma_{150} = \frac{409090.90}{150^2} + 18.18$$

$$= 36.36 \ N/mm^2$$

$$\sigma_{125} = \frac{409090.90}{125^2} + 18.18$$

$$= 44.36 \ N/mm^2$$

Inner cylinder:

The lame's equation for inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2 \rightarrow ⑤$$

$$\sigma_x = \frac{b_2}{x^2} + a_2 \rightarrow ⑥$$

Using boundary conditions

i) $x = r = 100mm$; $P_x = 0$

ii) $x = 125mm$; $P_x = P = 8 N/mm^2$

substitute boundary conditions

① $$8 = \frac{b_2}{125^2} - a_2 \rightarrow ⑦$$

② $$0 = \frac{b_2}{100^2} - a_2$$

$$a_2 = \frac{b_2}{100^2} \rightarrow ⑧$$

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substitute "a_2" value in eq (7)

$$8 = \left[\frac{b_2}{125^2} - \frac{b_2}{100^2}\right]$$

$$8 = b_2\left[\frac{1}{125^2} - \frac{1}{100^2}\right]$$

$$b_2 = -222222.22$$

substitute "b_2" value in eq (8)

$$a_2 = \frac{-222222.22}{100^2}$$

$$= -22.22$$

substitute a_2, b_2 values in eq (6)

the hoop stress in the cylinder inner $\sigma_x = \frac{b_2}{x^2} + a_2$

$$\sigma_{125} = \frac{-222222.22}{125^2} + (-22.22)$$

$$= -36.44 \text{ N/mm}^2$$

$$\sigma_{100} = \frac{-222222.22}{100^2} + (-22.22)$$

$$= -44.44 \text{ N/mm}^2$$

ii) stress due to after admiting the fluid pressure in the compound cylinder.

The lame's equations for compound cylinders :

$$P_x = \frac{b_3}{x^2} - a_3 \longrightarrow (9)$$

$$\sigma_x = \frac{b_3}{x^2} + a_3 \longrightarrow (10)$$

using boundary conditions

i) at $x = R^* = 150mm$; $P_x = 0$

ii) at $x = r^* = 100mm$; $P_x = P = 84.5 \ N/mm^2$

substitute boundary conditions in eq ⑨

$$① \Rightarrow 0 = \frac{b_3}{x^2} - a_3$$

$$0 = \frac{b_3}{150^2} - a_3$$

$$a_3 = \frac{b_3}{150^2} \longrightarrow ⑪$$

$$② \quad 84.5 = \frac{b_3}{100^2} - a_3 \rightarrow ⑫$$

substitute "a_3" value in eq ⑫

$$84.5 = \left[\frac{b_3}{100^2} - \frac{b_3}{150^2}\right]$$

$$84.5 = b_3\left[\frac{1}{100^2} - \frac{1}{150^2}\right]$$

$$b_3 = 1521000$$

substitute "b_3" value in eq ⑫

$$a_3 = \frac{1521000}{150^2} = 67.6$$

substitute "a_3", "b_3" values in eq ⑩

The hoop stress $\sigma_x = \frac{b_3}{x^2} + a_3$

$$R = \sigma_{150} = \frac{1521000}{150^2} + 67.6 = 135.2 \, N/mm^2$$

$$\gamma = \sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7 \, N/mm^2$$

$$r^* = \sigma_{125} = \frac{1521000}{125^2} + 67.6 = 164.944 \, N/mm^2$$

The resultant hoop stress will be algebric sum of due to shrinking before admiting and after.

a) for Inner cylinder

$$\sigma_{100} = -44.44 + 219.7 = 175.26 \, N/mm^2$$

$$\sigma_{125} = -36.44 + 164.94 = 120.5 \, N/mm^2$$

b) for outer cylinder

$$\sigma_{150} = 36.36 + 135.2 = 171.56 \, N/mm^2$$

$$\sigma_{125} = 44.36 + 164.94 = 209.3 \, N/mm^2$$

2) A compound cylinder is made by shrinkage. A cylinder of external diameter 200mm, internal Diameter 160mm. over the another cylinder of external Diameter 160mm, internal Diameter 120mm. The radial Pressure at the junction is $12.5 \, N/mm^2$. Find the resultant stress, when the compound cylinder is subjected to an internal fluid pressure of $60 \, N/mm^2$.

Sol) Given data

outer cylinder

External Diameter, "D" = 200mm

External Radious, "R" = $\frac{200}{2}$ = 100mm

Internal diameter "d" = 160 mm

Internal Radious "r" = 80mm

Inner cylinder

external Diameter, D^* = 160mm

External Radious R^* = 80mm

Internal Diameter d = 120mm

Internal Radious r = 60mm

pressure at Junction, P^* = 12.5 N/mm²

Internal fluid pressure, P = 60 N/mm²

i) The fluid before admiting the fluid pressure

a) outer cylinder:

The lame's equation for outer cylinder

$$P_x = \frac{b_1}{x^2} - a_1 \rightarrow (1)$$

$$\sigma_x = \frac{b_1}{x^2} + a_1 \rightarrow (2)$$

using boundary conditions

i) $x = R = 100mm$; $p_x = 0$

ii) $x = r^* = 80mm$; $p_x = P^* = 12.5 \, N/mm^2$

substitute boundary conditions in eq ①

① $0 = \frac{b_1}{100^2} - a_1$

$$a_1 = \frac{b_1}{100^2} \rightarrow ③$$

② $12.5 = \frac{b_1}{80^2} - a_1 \rightarrow ④$

substitute "a_1" value in eq ③ ④

$$12.5 = \left[\frac{b_1}{80^2} - \frac{b_1}{100^2}\right]$$

$$12.5 \; b_1 \left[\frac{1}{80^2} - \frac{1}{100^2}\right]$$

$$"b_1" = 222222.22$$

substitute "b_1" value eq ③

$$a_1 = \frac{222222.22}{100^2} = 22.22$$

substitute "a_1" "b_1" values in eq ②

The hoop stress $\sigma_x = \frac{b_1}{x^2} + a_1$

$$\sigma_{80} = \frac{222222.22}{80^2} + 22.22$$

$$= 56.94 \, N/mm^2$$

$$\sigma_{100} = \frac{222222.22}{100^2} + 22.22$$

$$= 44.44 \, N/mm^2$$

b) Inner cylinder:

The Lame's equation for inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2 \rightarrow (5)$$

$$\sigma_x = \frac{b_2}{x^2} + a_2 \rightarrow (6)$$

Using boundary conditions

i) $x = r^* = 80mm$; $P_x = 12.5 \, N/mm^2$

ii) $x = r = 60mm$; $P_x = 0$

substitute boundary conditions in eq (5)

$$(1) \Rightarrow 12.5 = \frac{b_2}{80^2} - a_2 \rightarrow (7)$$

$$(2) \Rightarrow 0 = \frac{b_2}{60^2} - a_2$$

$$a_2 = \frac{b_2}{80^2} \rightarrow (8)$$

substitute "a_2" value in eq (7)

$$12.5 = \left[\frac{b_2}{80^2} - \frac{b_2}{60^2}\right]$$

$$12.5 = b_2\left[\frac{1}{80^2} - \frac{1}{60^2}\right]$$

$$"b_2" = -102857.14$$

substitute "b_2" value in eq (8)

$$a_2 = \frac{-102857.14}{80^2}$$

$$= -28.57$$

substitute "a_2" "b_2" values in eq (6)

The hoop stress $\sigma_x = \frac{b_2}{x^2} + a_2$

$$\sigma_{60} = \frac{-102857.14}{60^2} + (-28.57)$$

$$= -57.14 \ N/mm^2$$

$$\sigma_{80} = \frac{-102857.14}{80^2} + (-28.57)$$

$$= -44.64 \ N/mm^2$$

ii) The fluid after admitted in the cylinder the lame's equation for compound cylinder.

$$P_x = \frac{b_3}{x^2} - a_3 \rightarrow (9)$$

$$\sigma_x = \frac{b_3}{x^2} + a_3 \rightarrow (10)$$

using boundary conditions

i) $x = R = 100mm$; $P_x = 0$

ii) $x = r = 60mm$; $P_x = P = 60 N/mm^2$

substitute boundary conditions in eq (9)

(1) $\Rightarrow$ $0 = \frac{b_3}{100^2} - a_3$

$$a_3 = \frac{b_3}{100^2} \rightarrow (11)$$

(2) $\Rightarrow 60 = \frac{b_3}{60^2} - a_3 \rightarrow$ (12)

substitute "a_3" value in eq (12)

$$60 = \left[\frac{b_3}{60^2} - \frac{b_3}{100^2}\right]$$

$$60 = b_3\left[\frac{1}{60^2} - \frac{1}{100^2}\right]$$

$$b_3 = 337500$$

substitute "b_3" value in eq (11)

$$a_3 = \frac{337500}{100^2} = 33.75$$

substitute "a_3" "b_3" values in eq (10)

The hoop stress $\sigma_x = \frac{b_3}{x^2} + a_3$

$$\sigma_{60} = \frac{337500}{60^2} + 33.75$$

$$= 127.5 N/mm^2$$

$$\sigma_{100} = \frac{337500}{100^2} + 33.75$$

$$= 67.5 N/mm^2$$

$$\sigma_{80} = \frac{337500}{80^2} + 33.75$$

$$= 86.48 N/mm^2$$

the algebric sum of hoop stress due shinkage before and after.

Outer cylinder :

$\sigma_{80} = 56.94 + 86.48 = 143.42\ N/mm^2$

$\sigma_{100} = 44.44 + 67.5 = 119.94\ N/mm^2$

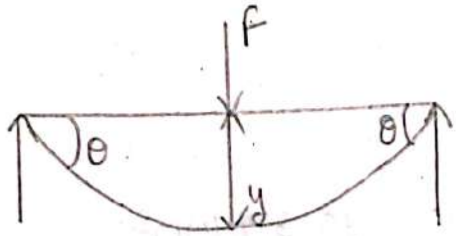
Inner cylinder :

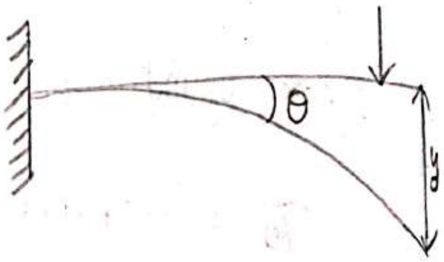
$\sigma_{60} = -57.14 + 12775 = 70.36\ N/mm^2$

$\sigma_{80} = -44.64 + 86.48 = 41.84\ N/mm^2$

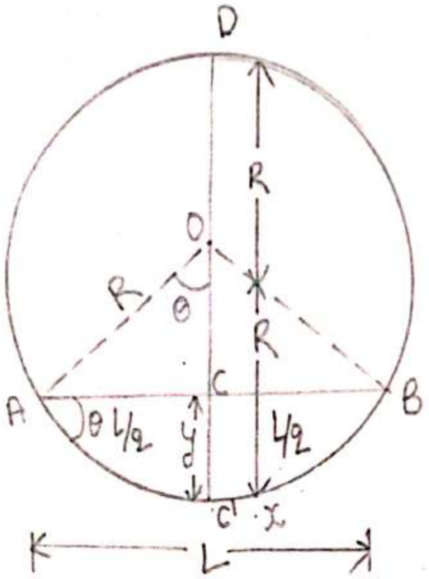
Unit-(4) Deflection of Beams

If a beam carries uniformly distributed load (or) a Point load, the beam is deflected from it's original position. Due to this, loads in the beam developed the bending moment.





Deflection and slope of a beam subjected to uniform BM



$DC' = 2R$

$DC = DC' - CC'$

$= 2R - y$

A beam AB of length "L" is subjected to a uniform bending moment "M". as the beam is subjected to constant BM, it will bend into circular arc,

→ The intial position of the beam is given by ACB, where as the deflected position is AC'B.

Let R = Radious of curvature

y = Deflection of Beam

θ = slope of the beam @ the end

In the geometry of a circle.

$$AC \times CB = DC \times CC'$$

$$L/2 \times L/2 = 2(R-y) \times y$$

$$L^2/4 = 2Ry - y^2$$

By neglecting the smaller values in the above equation.

$$L^2/4 = 2Ry$$

$$y = \frac{L^2}{8R} \rightarrow (1)$$

Based on Bending moment equation

$$\frac{m}{I} = \frac{E}{R}$$

$$R = \frac{EI}{m}$$

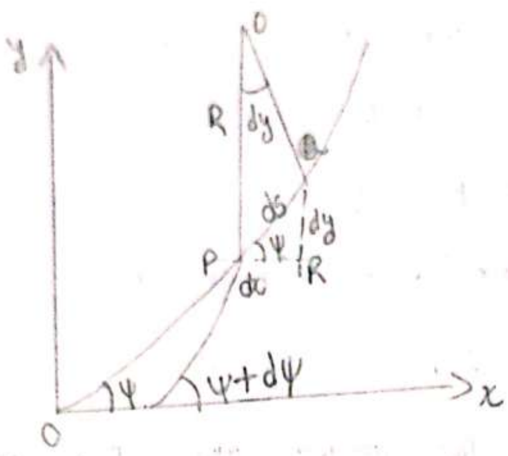
Substitute 'R' value in eq (1)

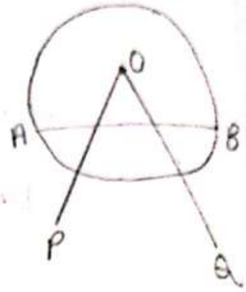
$$y = \frac{L^2}{8\left[\frac{EI}{m}\right]} = \frac{mL^2}{8EI} \rightarrow (2)$$

from $\triangle ADC$, $\sin\theta = \frac{AC}{OA} = \frac{L/2}{R} = \frac{L}{2R}$

$$\sin\theta \simeq \theta = \frac{L}{2R} = \frac{L}{2\left[\frac{EI}{m}\right]} = \frac{mL}{2EI}$$

Relation B/w slope, Deflection and Radius of curvature





Let the curve AC'B represents the deflection of Beam

consider a small portion PQ on the beam.

Draw the Tangent at P & Q with an angle of ψ and $(\psi + d\psi)$ with x-axis.

Draw the normal lines at "P" and "Q" will meet at point O.

$$PO = QO = R$$

let the length of the arc or curve $PQ = ds$

$$d\psi = \frac{ds}{dR} \Rightarrow R\,d\psi = ds$$

$$R = \frac{ds}{d\psi} \rightarrow ③$$

In, the Diagram, ΔPQR, $\tan\psi = \frac{dy}{dx} \rightarrow ④$

$$\sin\psi = \frac{dy}{ds}$$

$$\cos\psi = \frac{dx}{\ldots}$$

From eq ③ $\Rightarrow R = \frac{ds}{d\psi}$

$R = \frac{ds/\ldots}{d\ldots}$

$R = \ldots$

$R = \ldots$

Differentiate the ...

Tan ψ ...

$\frac{d}{dx}$ ...

substitute ...

from eq ③ $\Rightarrow R = \frac{ds}{d\psi}$

$$R = \frac{ds/dx}{d\psi/dx}$$

$$R = \frac{1/\cos\psi}{d\psi/dx}$$

$$R = \frac{\sec\psi}{d\psi/dx} \rightarrow ⑤$$

Differentiate the eq ④

$$\tan\psi = \frac{dy}{dx}$$

$$\frac{d}{dx}(\tan\psi) = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$\sec^2\psi \times \frac{d\psi}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{d\psi}{dx} = \frac{1}{\sec^2\psi} \cdot \frac{d^2\psi}{dx^2}$$

substitute the above value in eq ⑤

$$R = \frac{\sec\psi}{\frac{1}{\sec^2\psi} \times \frac{d^2y}{dx^2}} = \frac{\sec^3\psi}{\frac{d^2y}{dx^2}}$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{(\sec^2\psi)^{3/2}} \quad \left\{\begin{array}{l} \sec^2\theta - \tan^2\theta = 1 \\ \sec^2\theta = 1+\tan^2\theta \end{array}\right\}$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{(1+\tan^2\psi)^{3/2}}$$

Assume ψ is small angle, $\tan^2\psi$ is also very small angle, neglecting small terms.

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

from Bending equation.

$$\frac{m}{I} = \frac{E}{R}$$

$$\frac{m}{EI} = \frac{1}{R}$$

$$\frac{m}{EI} = \frac{d^2y}{dx^2}$$

$$\boxed{m = EI\frac{d^2y}{dx^2}} \rightarrow (6)$$

Differentiate eq (6) w.r to "x"

$$\frac{dm}{dx} = EI\frac{d^3y}{dx^3}$$

$$\boxed{F = EI\frac{d^3y}{dx^3}} \rightarrow (7)$$

Differentiate eq (7) w.r to "x"

$$\frac{dF}{dx} = EI\frac{d^4y}{dx^4}$$

$$\boxed{w = EI\frac{d^4y}{dx^4}}$$

Deflection $= y$

Slope "θ" $= \frac{dy}{dx}$

Bm "m" $= EI\frac{d^2y}{dx^2}$

Force "F" $= EI\frac{d^3y}{dx^3}$

load "w" $= EI\frac{d^4y}{dx^4}$

Bm = Bending moment

w = Rate of loading

Double integration method:

Deflection of a cantilever with a point load at the free end:

A cantilever AB of length "L" fixed at point "A" and free at point B.

consider a section x-x at a distance "x" from the end A. The Beam @ this section.

$$m_x = -W(L-x) \rightarrow ①$$

generally Bending moment equation

$$EI \frac{d^2y}{dx^2} = m$$

$$EI \frac{d^2y}{dx^2} = -W(L-x)$$

$$EI \frac{d^2y}{dx^2} = -WL + Wx$$

Integrating the above equation

$$\int EI \frac{d^2y}{dx^2} = \int WL + \int Wx$$

$$EI \frac{dy}{dx} = -WLx + \frac{Wx^2}{2} + \int C_1 \rightarrow ②$$

Integrating above equation

$$\int EI\frac{dy}{dx} = \int -wlx + \int \frac{wx^2}{2} + \int C_1$$

$$EI \times y = \frac{-wlx}{2} + \frac{wx^3}{6} + C_1x + C_2 \rightarrow (3)$$

Boundary conditions:

i) At A; $x=0$, $y=0$

ii) At B; $x=0$, $\frac{dy}{dx}=0$

Substitute Boundary conditions in eq (2) & (3)

$$EI \times 0 = 0+0+0+C_2$$

$$\boxed{C_2=0}$$

$$EI \times 0 = 0+0+0+C_1$$

$$\boxed{C_1=0}$$

Substitute "C_1", "C_2" values in eq (2) & (3)

$$EI\frac{dy}{dx} = -wlx + \frac{wx^2}{2} \rightarrow (4)$$

Eq (4) is known as slope equation.

At end A; slope and deflection is zero.

At end B; $x=L$

$$(4) \Rightarrow EI\frac{dy}{dx} = -wL \times L + \frac{wL^2}{2}$$

$$EI\frac{dy}{dx} = -wL^2 + \frac{wL^2}{2}$$

$$= \frac{-2wL^2 + wL^2}{2}$$

$$EI\frac{dy}{dx} = \frac{-wL^2}{2}$$

$$\frac{dy}{dx} = \frac{-wL^2}{2EI}$$

$$\boxed{\theta = \frac{wL^2}{2EI} \text{ Downward}}$$

$$③ \Rightarrow EIy = \frac{-wLx^2}{2} + \frac{wx^3}{6} + C_1x + C_2$$

$$EIy = \frac{-wLx^2}{2} + \frac{wx^3}{6} \rightarrow ⑤$$

Eq ⑤ is known as deflection.

At end B; $x = L$

$$EI \times y = \frac{-wL \times L^2}{2} + \frac{wL^3}{6}$$

$$= \frac{-wL^3}{2} + \frac{wL^3}{6}$$

$$= \frac{-3wL^2 + wL^3}{6}$$

$$EIxy = \frac{-2wL^3}{6}$$

$$y = \frac{-wL^3}{3EI}$$

$$\boxed{y = \frac{wL^3}{3EI} \text{ (Downward)}}$$

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Deflection of a cantilever with a UDL :

A cantilever AB of length "L"

is fixed at one end "A" and

free at "B".

consider an x-x section at a distance of "x" from end

the Beam will be,

$$m = \frac{-w(L-x)^2}{2}$$

from BM equation, we have

$$m = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = \frac{-w(L-x)^2}{2}$$

Intigrating the above equation

$$\int EI \frac{d^2y}{dx^2} = \int \frac{-w(L-x)^2}{2}$$

$$EI \frac{dy}{dx} = \frac{-w(L-x)^3}{6}(-1) + C_1$$

$$EI \frac{dy}{dx} = \frac{w(L-x)^3}{6} + C_1 \rightarrow 1$$

Integrating the above equation

$$EI\, xy = -\frac{w(L-x^4)}{24} + C_1 x + C_2 \rightarrow 2$$

Boundary conditions:

i) At A; $x=0$, $y=0$

ii) At B; $x=0$ $\frac{dy}{dx}=0$

substitute BC in eq ① and ②

$$\text{eq ①} \Rightarrow EI(0) = \frac{w(L-0)^3}{6} + C_1$$

$$0 = \frac{wL^3}{6} + C_1$$

$$C_1 = \frac{-wL^3}{6}$$

$$\text{eq ②} \Rightarrow 0 = \frac{-w(L-0)^4}{24} + C_1(0) + C_2$$

$$C_2 = \frac{wL^4}{24}$$

Substitute "C_1" and "C_2" values in eq ① & ②

$$① \Rightarrow EI\frac{dy}{dx} = \frac{w(L-x)^3}{6} - \frac{wL^3}{6} \rightarrow ③$$

Eq ③ is known as slope equation

At end B, $x=L$

$$EI\frac{dy}{dx} = \frac{w(L-L)^3}{6} - \frac{wL^3}{6}$$

$$EI\frac{dy}{dx} = \frac{-wL^3}{6}$$

$$\frac{dy}{dx} = \frac{-wL^3}{6EI} \qquad \boxed{\theta = \frac{-wL^3}{6EI}}$$

② $\Rightarrow EI\,xy = \frac{-w(L-L)}{24} + C_1 L + C_2$

$$EIy = 0 - \frac{wL^3}{6}L + \frac{wL^4}{24}$$

$$EIy = \frac{-wL^4}{6} + \frac{wL^4}{24}$$

$$EIy = \frac{-4wL^4 + wL^4}{24}$$

$$EIy = \frac{-\overset{1}{\cancel{3}}wL^4}{\underset{8}{\cancel{24}}EI}$$

$$\boxed{y = \frac{-wL^4}{8EI}}$$

moment area method

Deflection and slope of a cantilever with point load at free end :

A cantilever of length "L" fixed at "A" and free at the end "B".

The Beam will be zero at free end and wt at fixed end "A".

At the fixed end "A" the slope and Deflection are zero.

According to moment area method,

Slope at B; $\theta_B = \frac{A}{EI} = \frac{\text{Area of Bending moment Diagram}}{EI}$

$$= \frac{wl^2/2 \times \frac{L}{3}}{EI}$$

$$\boxed{y_B = \frac{wl^3}{3EI}}$$

Deflection and slope of a cantilever with UDL:

slope at $\theta_B = \frac{A}{EI}$

$$= \frac{\frac{1}{3} \times \frac{wl^2}{2} \times L}{EI}$$

$$\boxed{\theta_B = \frac{wL^3}{6EI}}$$

Deflection at B, $y_B = \frac{A\bar{x}}{EI}$

$$= \frac{wl^2/_{2l} \times \frac{3l}{4}}{EI}$$

$y_B = \frac{wl^3}{\ldots}$

Simply supported Beam

At ends "A" and "B" BM is zero and max at the

Slope at c, $\theta_C = \frac{A}{EI}$

$$= \frac{\frac{1}{2} \times \frac{L}{2} \times \frac{wL}{4}}{EI}$$

$$\boxed{\theta_C = \frac{wL^2}{16EI}}$$

Deflection at c, $y_C = \frac{A\bar{x}}{EI}$

$$\frac{\frac{wL^2}{16} \times \frac{\cancel{2}L/\cancel{2}}{3}}{EI}$$

$$\boxed{= \frac{wL^3}{48EI}}$$